

A Light Intro To Boosting

Machine Learning

- Not as cool as it sounds
 - Not iRobot
 - Not Screamers (no Peter Weller 😞)
- Really just a form of
 - Statistics
 - Optimization
 - Probability
 - Control theory
 - ...
- We focus on classification

Classification

- A subset of machine learning & statistics
- Classifier takes input and predicts the output
- Make a classifier from a *training* dataset
- Use the classifier on a *test* dataset (different from the training dataset) to make sure you didn't just memorize the training set
- A good classifier will have low *test* error

Classification and Learning

- Learning classifier learns how to predict after being shown many input-output examples
- Weak classifier is slightly correlated with correct output
- Strong classifier is highly correlated with correct output
- (See the PAC learning model for more info)

Methods for Learning Classifiers

- Many methods available
 - Boosting
 - Bayesian networks
 - Clustering
 - Support Vector Machines (SVMs)
 - Decision Trees
 - . . .
- We focus on boosting

Boosting

- Question: Can we take a bunch of weak hypotheses and create a very good hypothesis?
- Answer: Yes!

Brief History of Boosting

- 1984 - Framework developed by Valiant
 - Probably approximately correct (PAC)
- 1988 - Problem proposed by Michael Kearns
 - Machine learning class taught by Ron Rivest
- 1990 - Boosting problem solved (in theory)
 - Schapire, recursive majority gates of hypotheses
 - Freund, simple majority vote over hypotheses
- 1995 - Boosting problem solved (in practice)
 - Freund & Schapire, AdaBoost adapts to error of hypotheses

T Weak Hyps = 1 Strong Hyp

**Try Many
Weak Hyps**

Weak Hyp

Weak Hyp

Weak Hyp

Weak Hyp

Weak Hyp

Weak Hyp

Weak Hyp

Weak Hyp

Weak Hyp

Weak Hyp

Weak Hyp

T Weak Hyps = 1 Strong Hyp

Try Many
Weak Hyps

Weak Hyp

Weak Hyp

Weak Hyp

Weak Hyp

Weak Hyp

Weak Hyp

Weak Hyp

Weak Hyp

Weak Hyp

Weak Hyp

Weak Hyp

Weak Hyp

Combine T Weak Hyps

Weight 1

Weak Hyp 1

+

Weight 2

Weak Hyp 2

+

•

•

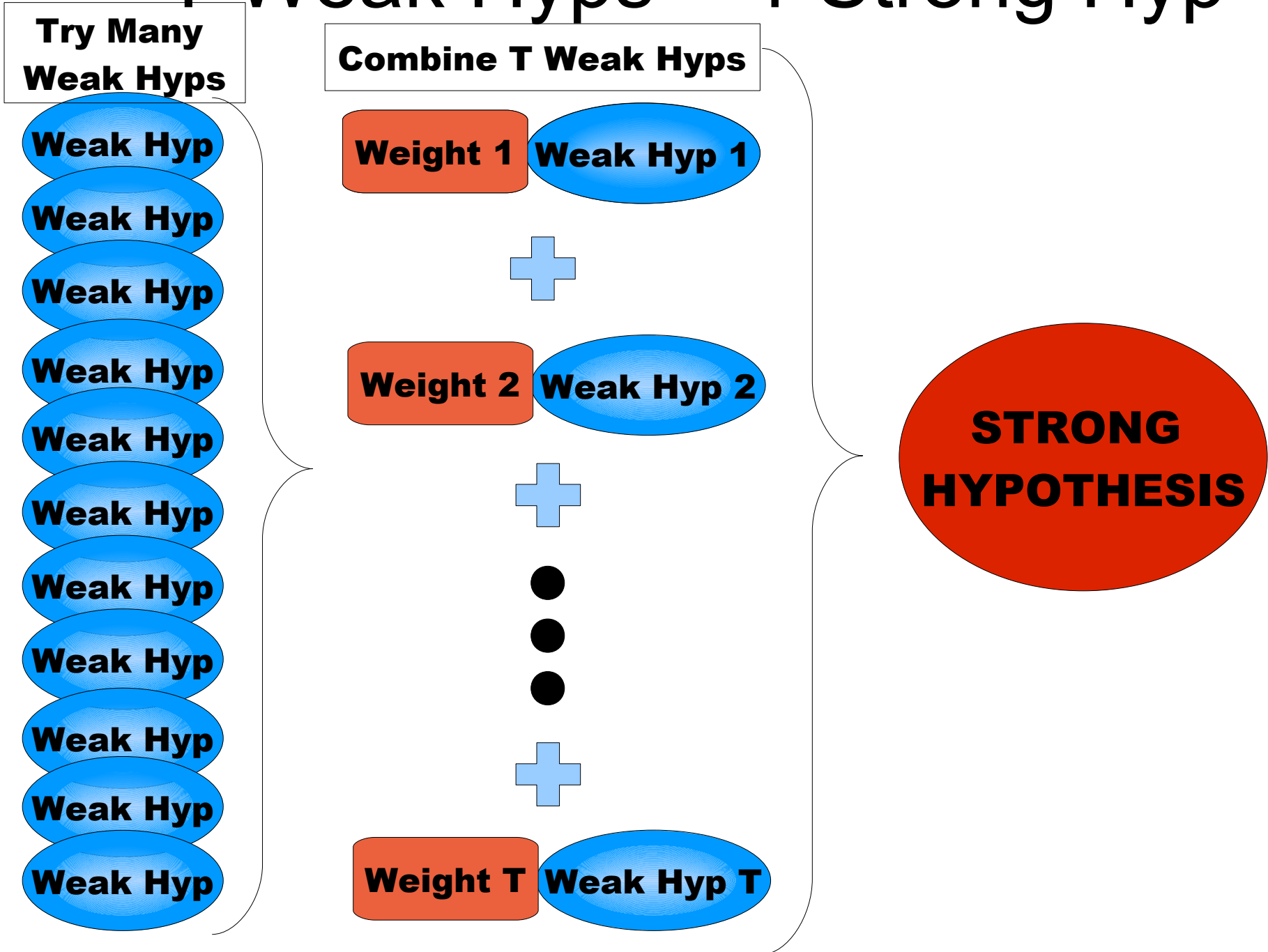
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+

Weight T

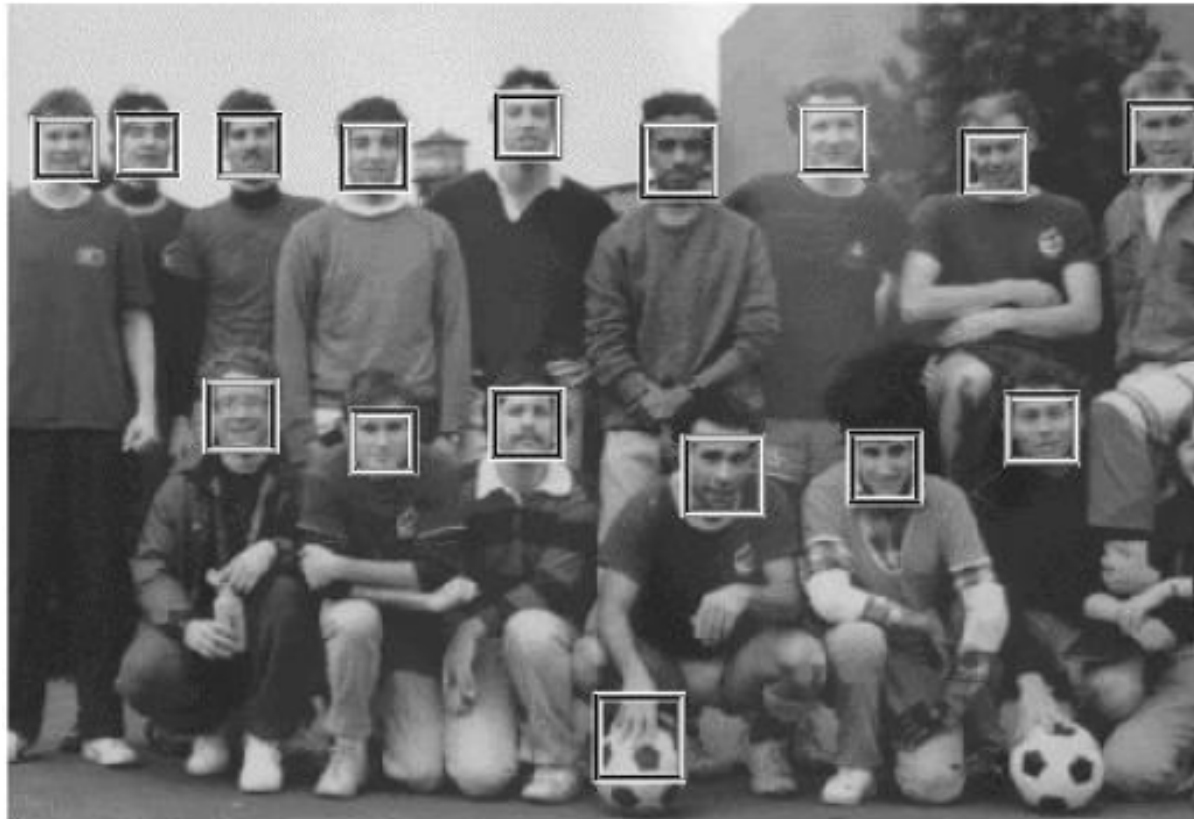
Weak Hyp T

T Weak Hyps = 1 Strong Hyp



Example: Face Detection

- We are given a dataset of images
- We need to determine if there are faces in the images



Example: Face Detection

- Go through each possible rectangle
- Some weak hypotheses might be:
 - Is there a round object in the rectangle?
 - Does the rectangle have darker spots where the eyes should be?
 - Etc.
- Classifier = $2.1 * (\text{Is Round}) + 1.2 * (\text{Has Eyes})$
- Viola & Jones 2001 solved face detection problem in similar manner

Algorithms

- Many boosting algorithms have two sets of weights
 - Weights on all the training examples
 - Weights for each of the weak hypotheses used
- It is usually clear from context which set of weights is being discussed

Basic Boosting Algorithm

- Initial Conditions:
 - Training dataset $\{ (x_1, y_1), \dots (x_i, y_i) \dots, (x_n, y_n) \}$
 - Each x is an example with a label y
- Learn a pattern
 - Use T weak hypotheses
 - Combine them in an “intelligent” manner
- See how well we learned the pattern
 - Did we just memorize training set?

An Iterative Learning Algorithm

Let w_i^t be the weight of example i on round t

$$w_i^0 = 1/n$$

For $t = 1$ to T :

- 1) Try many weak hyps, compute error $\sum_i w_i^t [[h(x_i) \neq y_i]]$
- 2) Pick the best hypothesis: h_t
- 3) Give h_t a weight α_t
- 4) More weight to examples that h_t misclassified
- 5) Less weight to examples that h_t classified correctly

Return a final hypothesis of $H_t(x) = \sum_t \alpha_t h_t(x)$

One Iteration

Dataset

A diagram illustrating the input to a single neuron. On the left, a yellow oval contains the weight vector w_i^t . On the right, a green oval contains the input-output pair x_i, y_i .

w_i^t x_i, y_i

A diagram illustrating the input to a linear model. On the left, a yellow box contains the weight vector w_i^t . On the right, a green oval contains the input data point x_i, y_i .



w_i^t x_i, y_i



w^t x_i, y_i

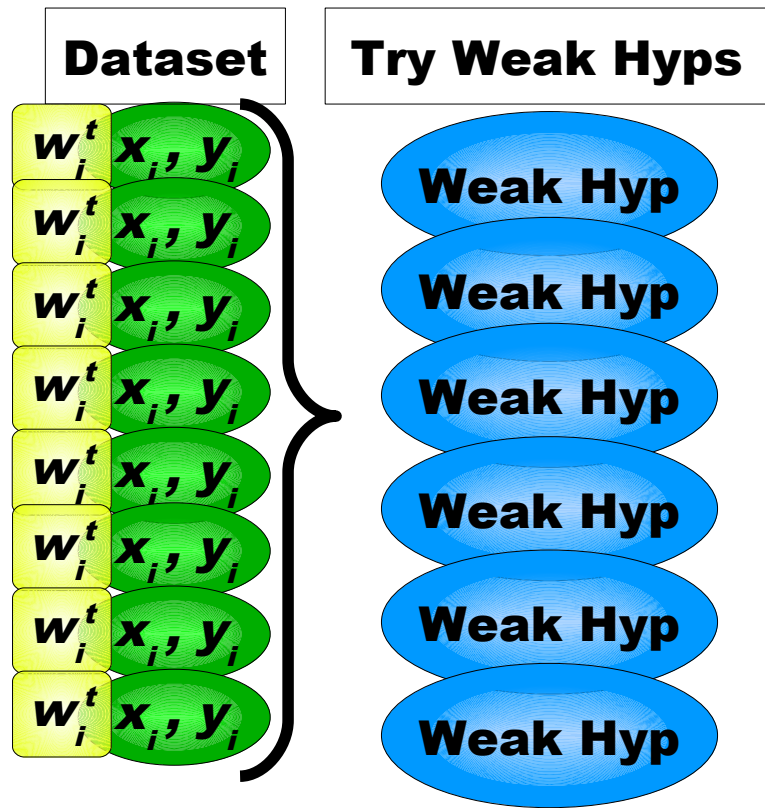
w_i^t x_i, y_i



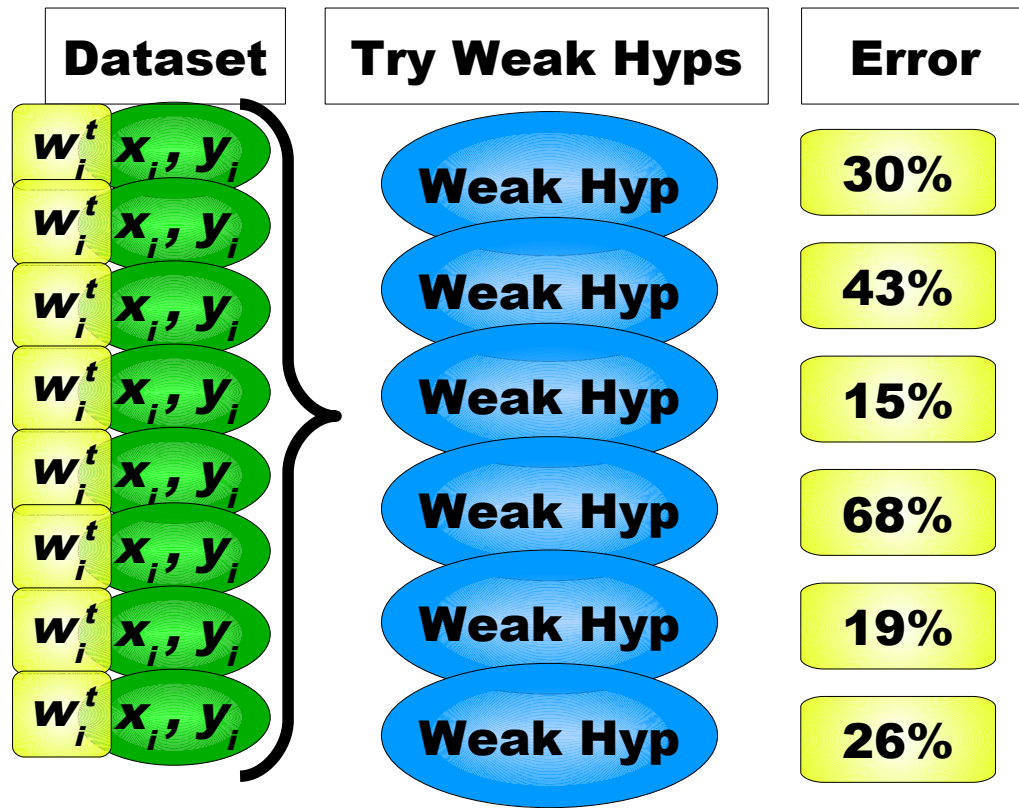
w_i^t x_i, y_i

A diagram illustrating the input to a neural network layer. It consists of a yellow box on the left containing the text w^t and a green oval on the right containing the text x_i, y_i .

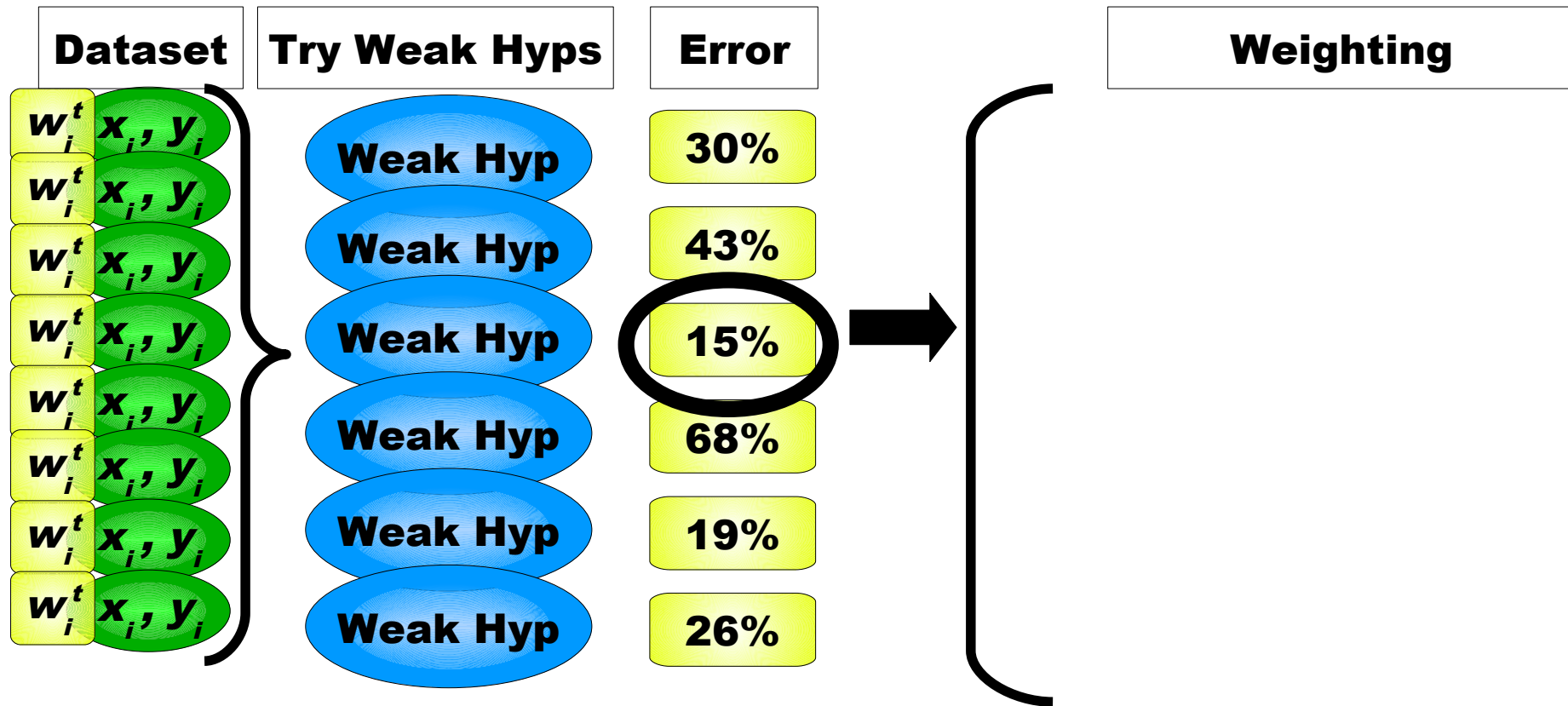
One Iteration



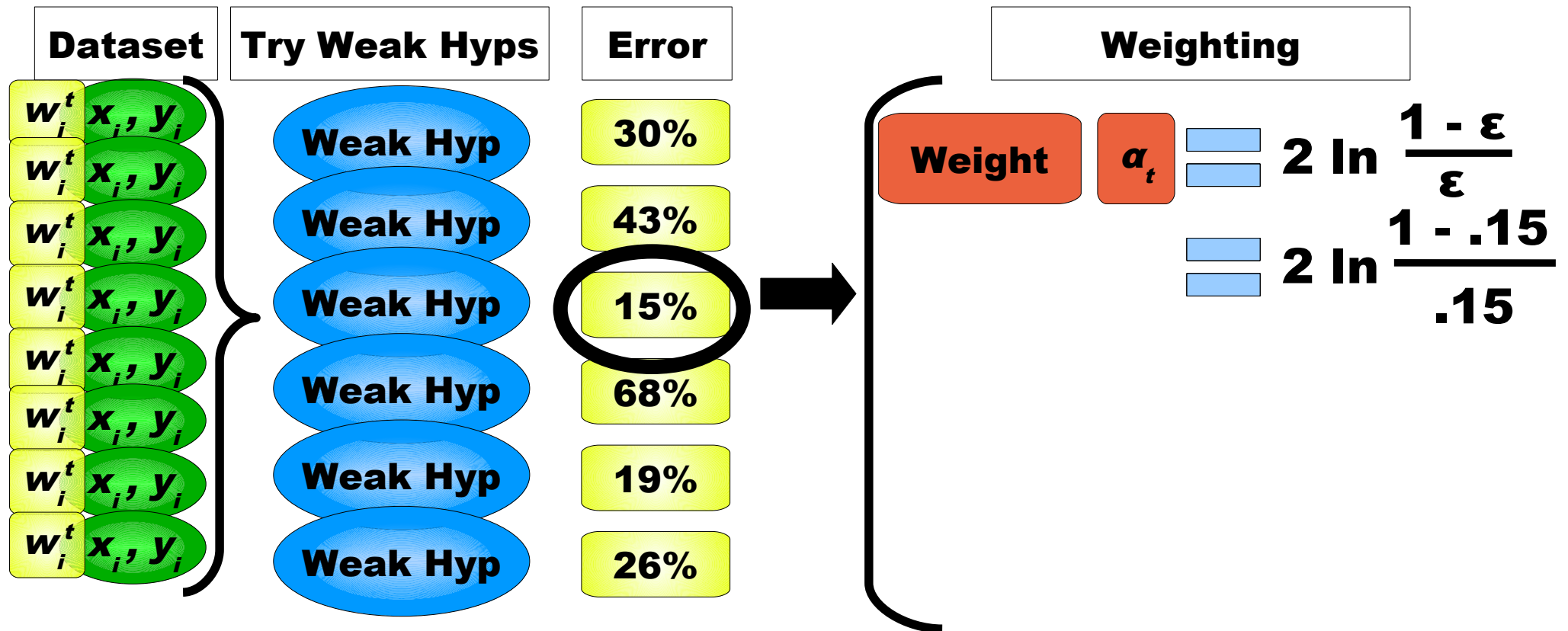
One Iteration



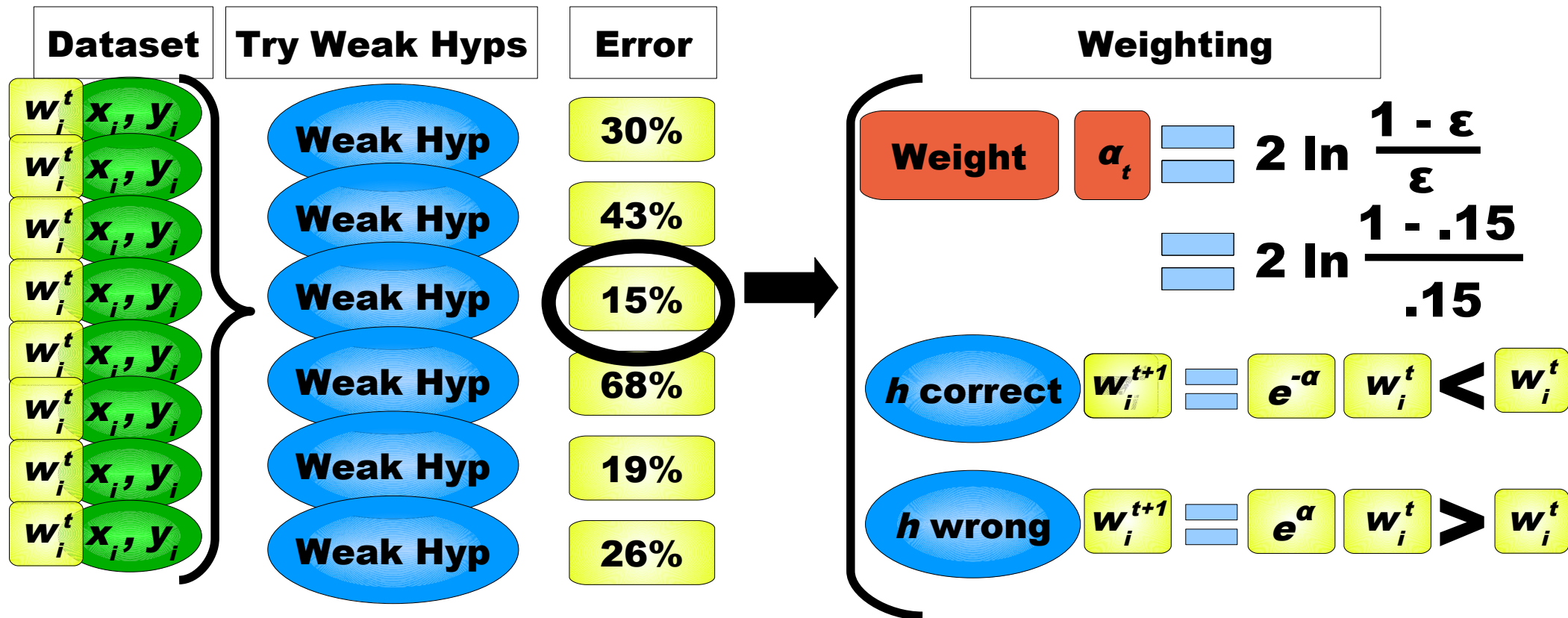
One Iteration



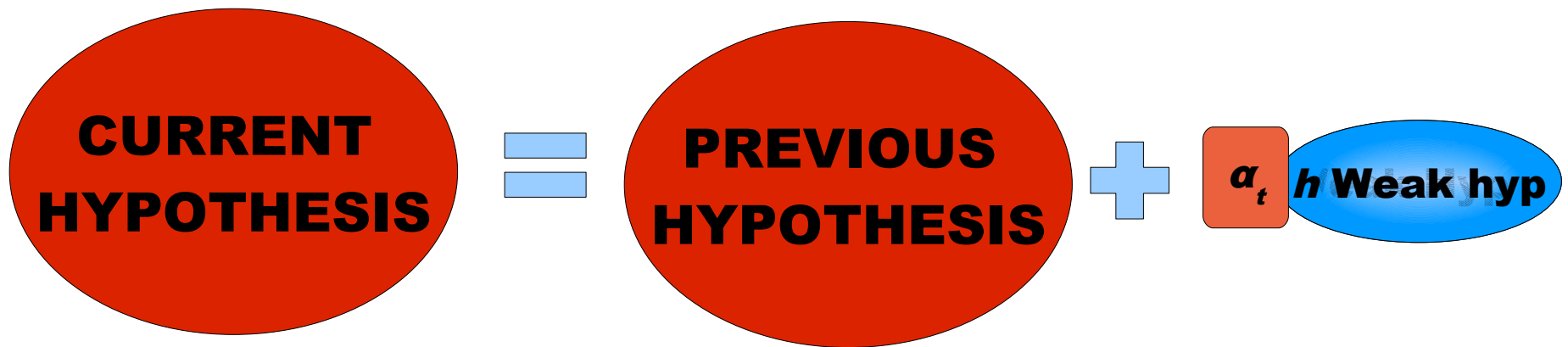
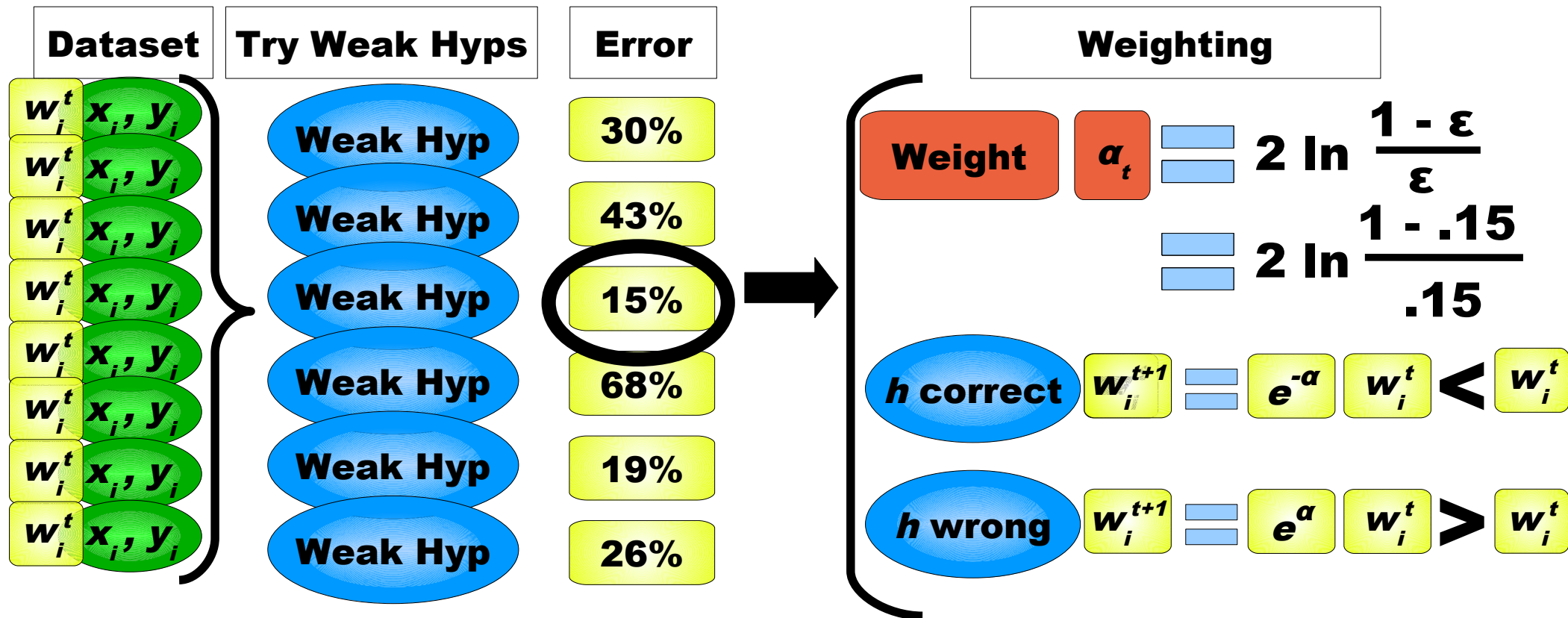
One Iteration



One Iteration

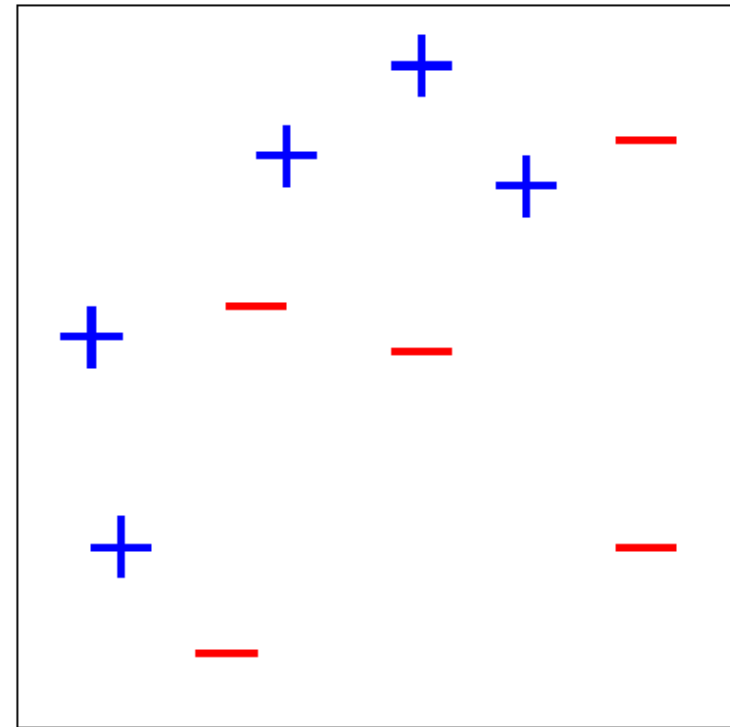


One Iteration

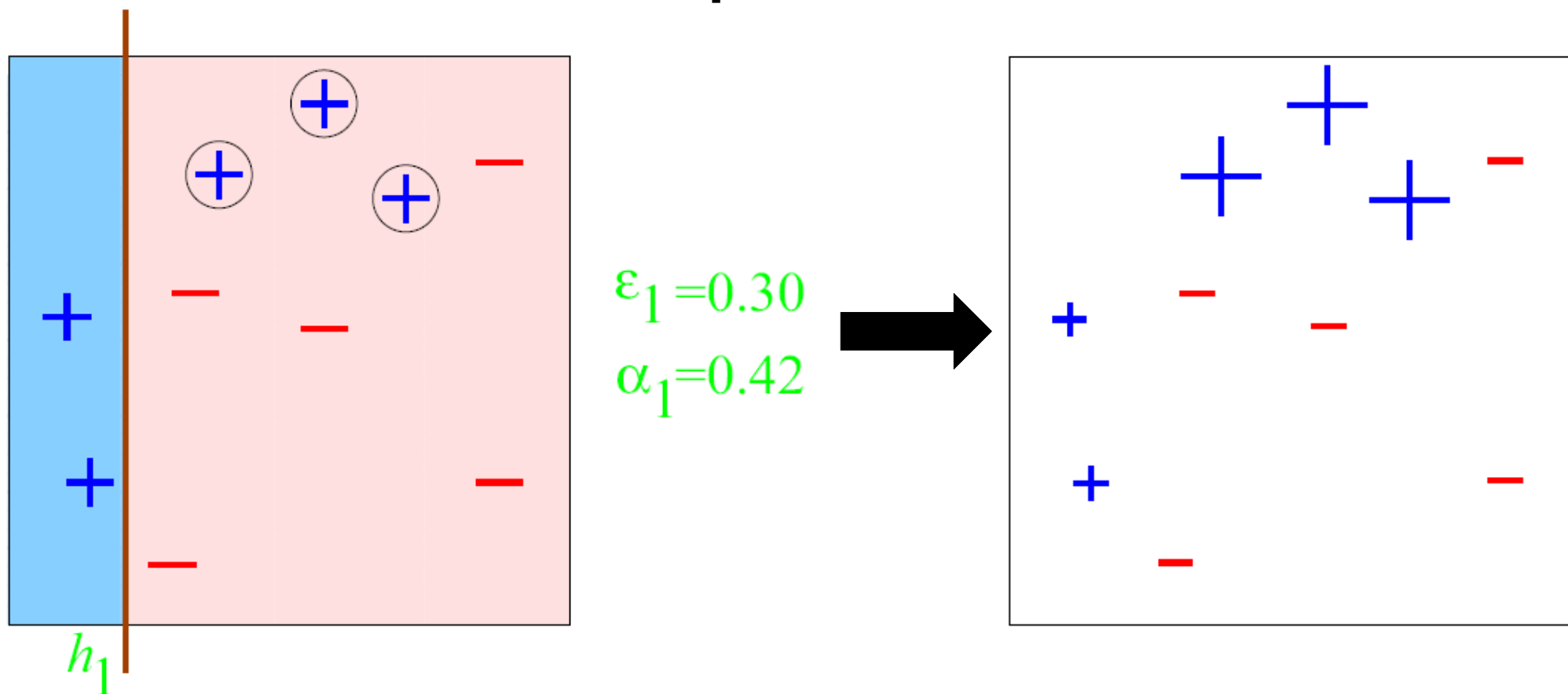


Toy Example

- Positive examples
- Negative examples
- 2-Dimensional plane
- Weak hyps: linear separators
- 3 iterations

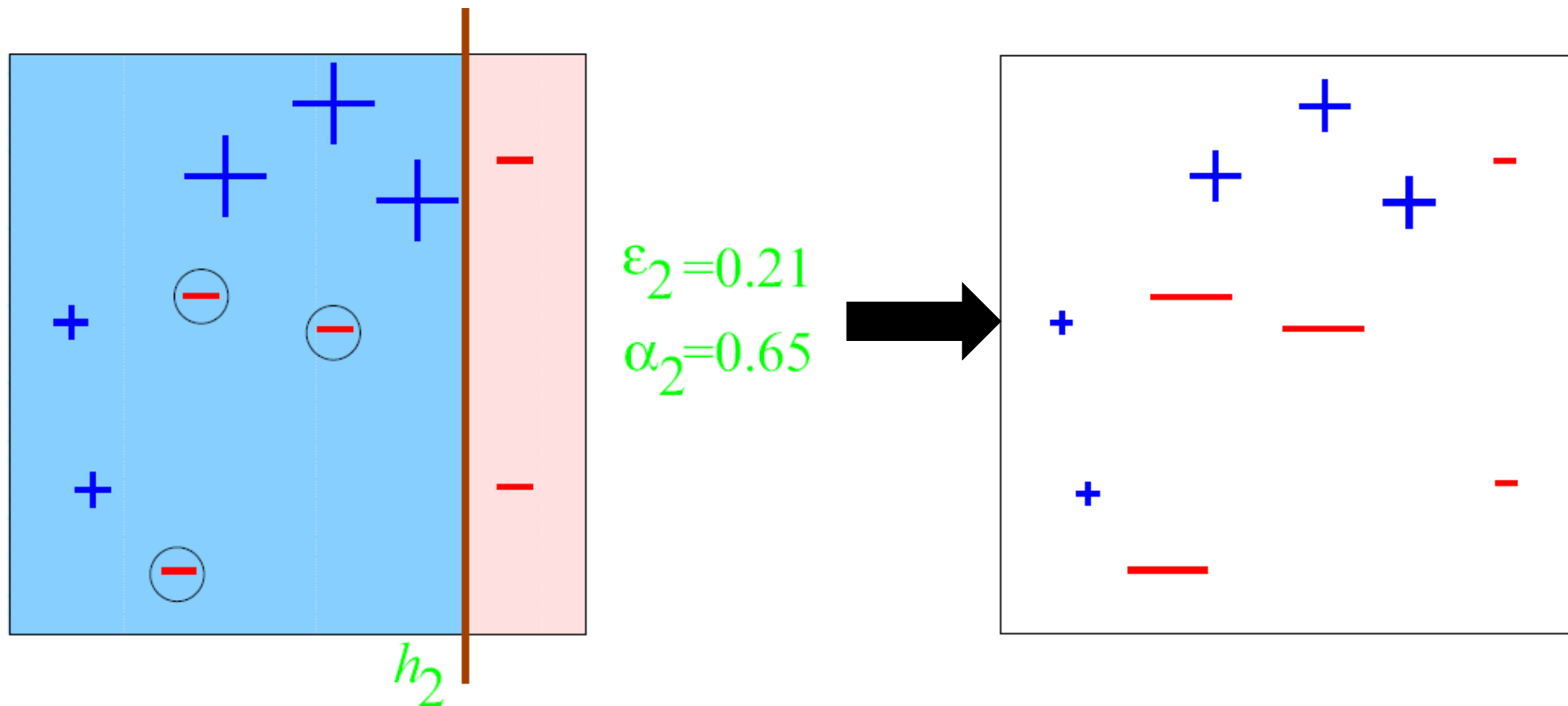


Toy Example: Iteration 1



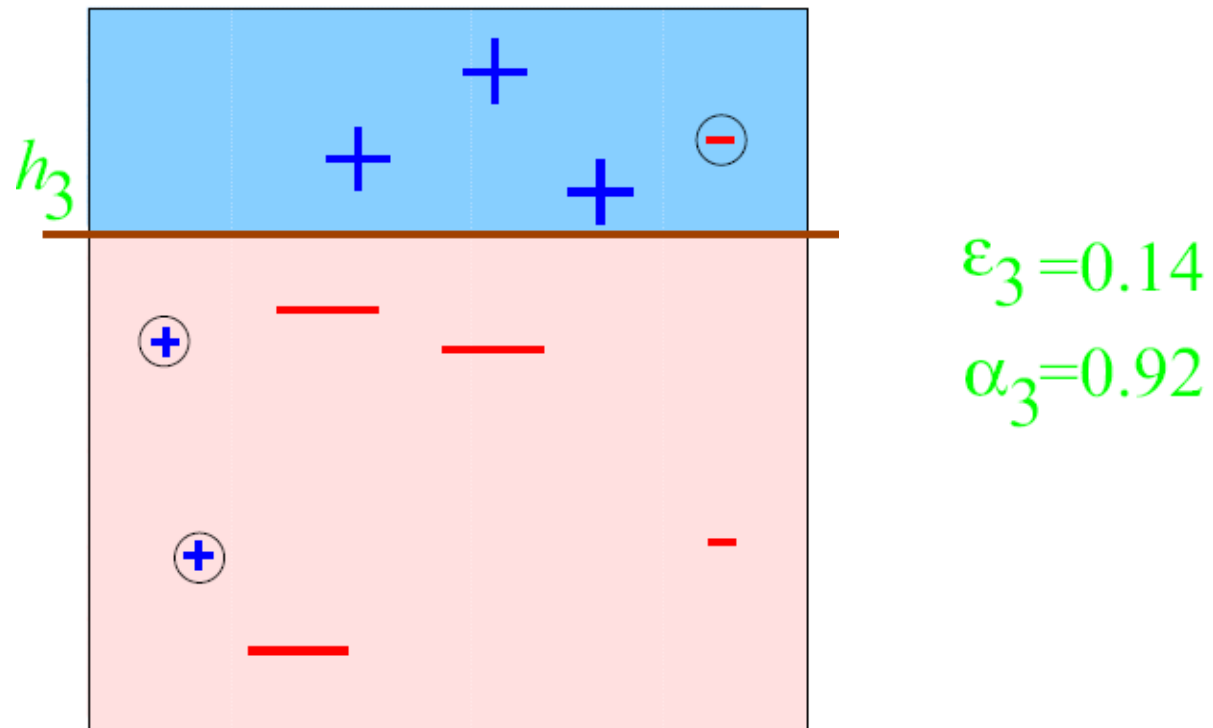
Misclassified examples are circled, given more weight

Toy Example: Iteration 2



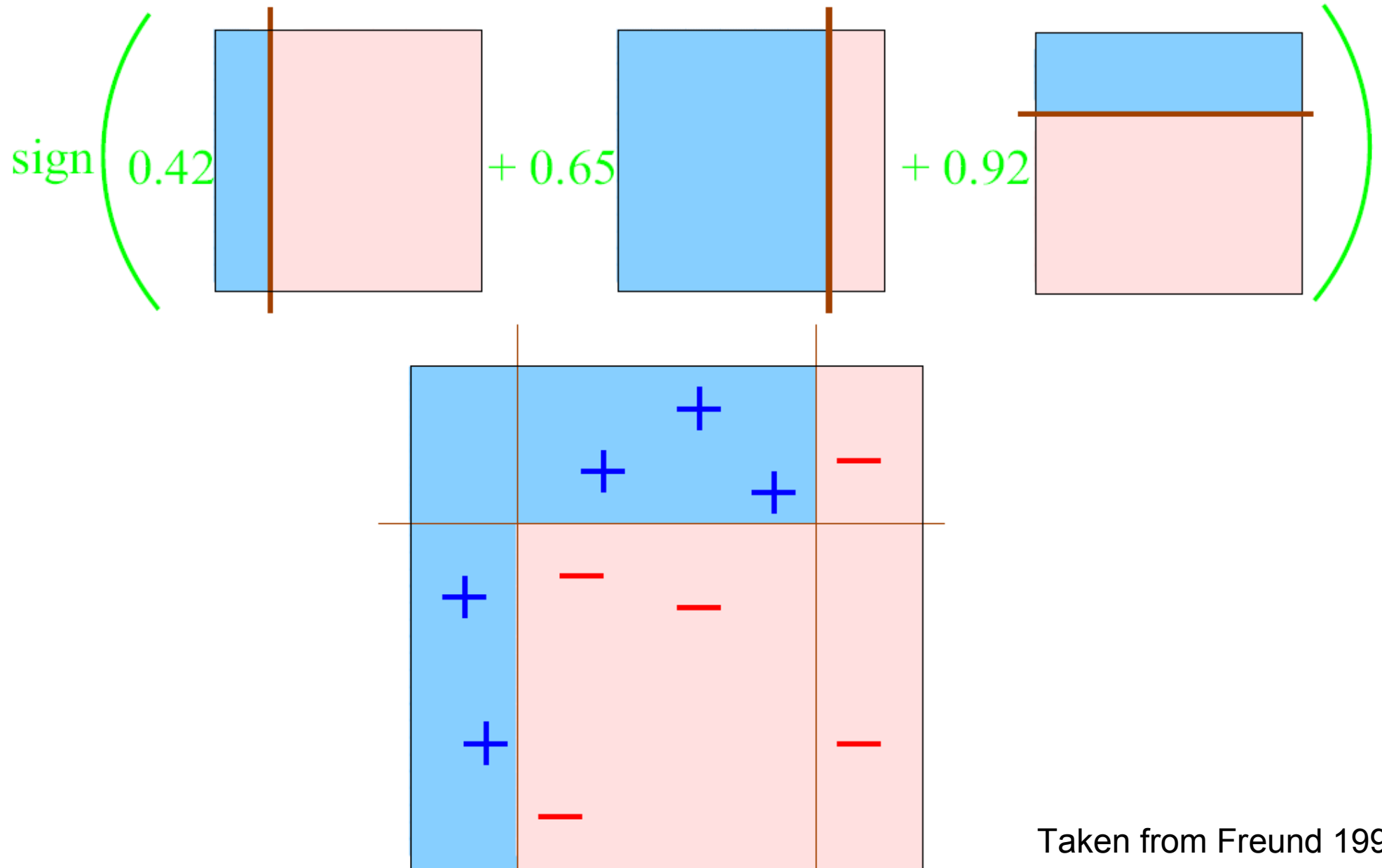
Misclassified examples are circled, given more weight

Toy Example: Iteration 3



Finished boosting

Toy Example: Final Classifier



Questions

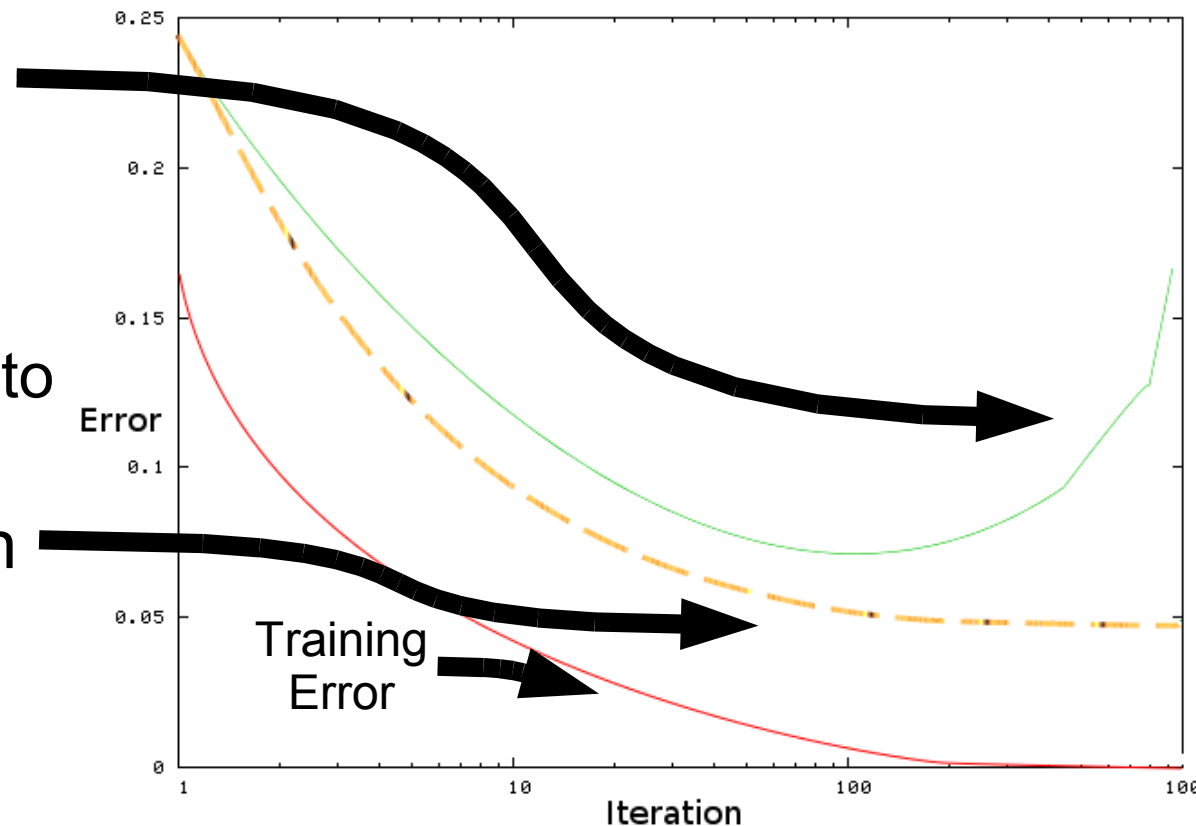
- How should we weight the hypotheses?
- How should we weight the examples?
- How should we choose the “best” hypothesis?
- How should we add the new (this iteration) hypothesis to the set of old hypotheses
- Should we consider old hypotheses when adding new ones?

Answers

- There are many answers to these questions
- Freund & Schapire 1997 – AdaBoost
- Schapire & Singer 1999 – Confidence rated AdaBoost
- Freund 1995, 2000 – Noise resistant via binomial weights
- Friedman *et al* 1998 and Collins *et al* 2000 – Connections to logistic regression and Bregman divergences
- Warmuth *et al* 2006 – “Totally corrective” boosting
- Freund & Arvey 2008 – Asymmetric cost, boosting the normalized margin

What's the big deal?

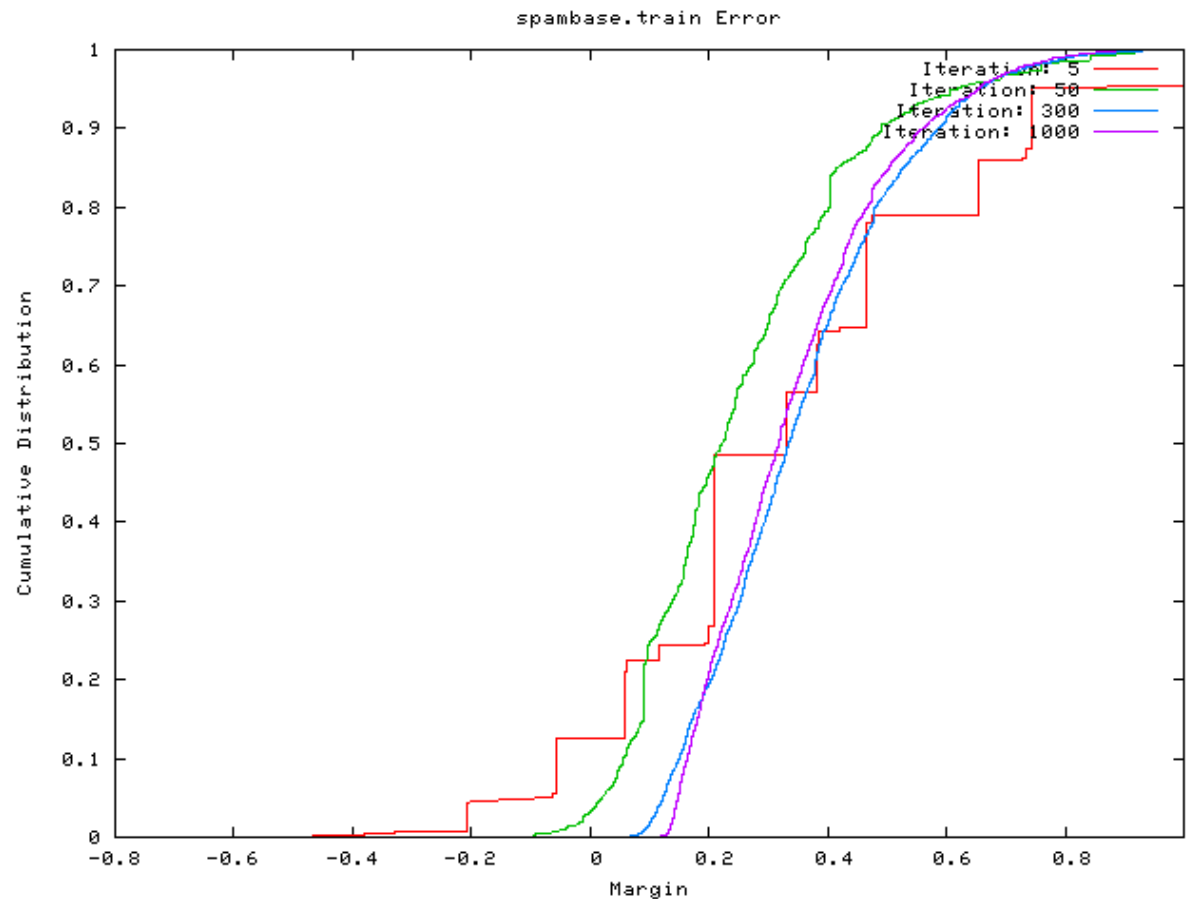
- Most algorithms start to memorize the data instead of learning patterns
- Most test error curves
 - Train decreases
 - Test starts to increase
 - Increase in test is due to “overfitting”
- Boosting continues to learn
 - Test error plateaus
- Explanation: margin



What's the big deal?

- One goal in machine learning is “margin”
 - “Margin” is a measure of how correct an example is
 - If all hypotheses get an example right, we'll probably get a similar example right in the future
 - If 1 out of 1000 hypotheses get an example right, then we'll probably get it wrong in the future
 - Boosting gives us a good margin

Margin Plot



- Margin frequently converges to some cumulative distribution function (CDF)
- Rudin et al. show that CDF may not always converge

End Boosting Section

Start Final Classifier Section

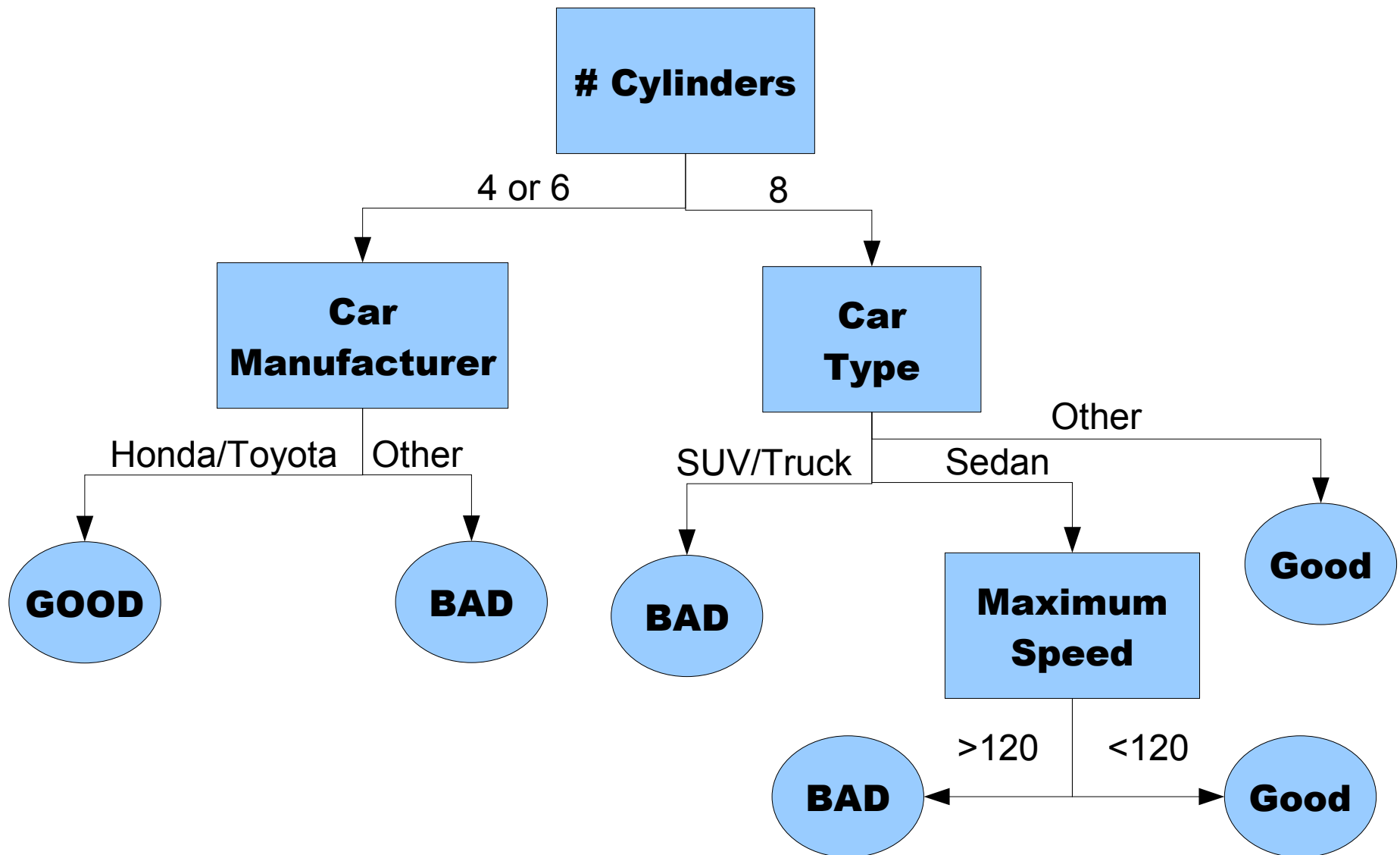
Final Classifier: Combination of Weak Hypotheses

- Original usage of boosting was just adding many weak hypotheses
- Adding weak hyps could be improved
 - Some of the weak hypotheses may be correlated
 - If there are a lot of weak hypotheses, the decision can be very hard to visualize
- Why can't boosting be more like decision trees
 - Easy to understand and visualize
 - A classic approach used by many fields

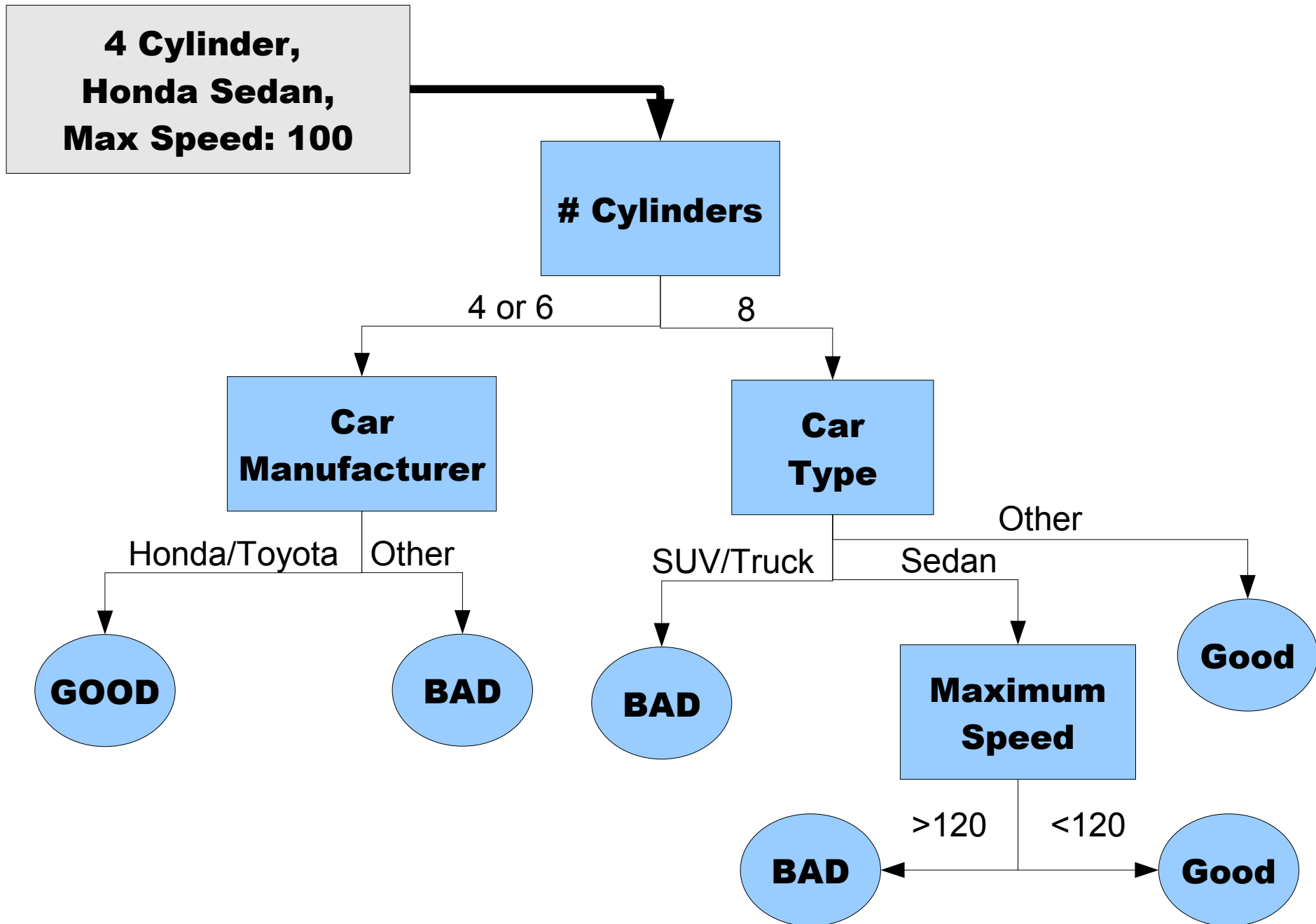
Final Classifier: Decision Trees

- Follow a series of questions to a single answer
- Does the car have 4 or 8 cylinders?
 - If #cylinders=4 or 8, then was the car made in Asia?
 - If Yes then you get good gas mileage
 - If no then you get bad gas mileage
 - If #cylinders=3,5,6, or 7 then poor gas mileage

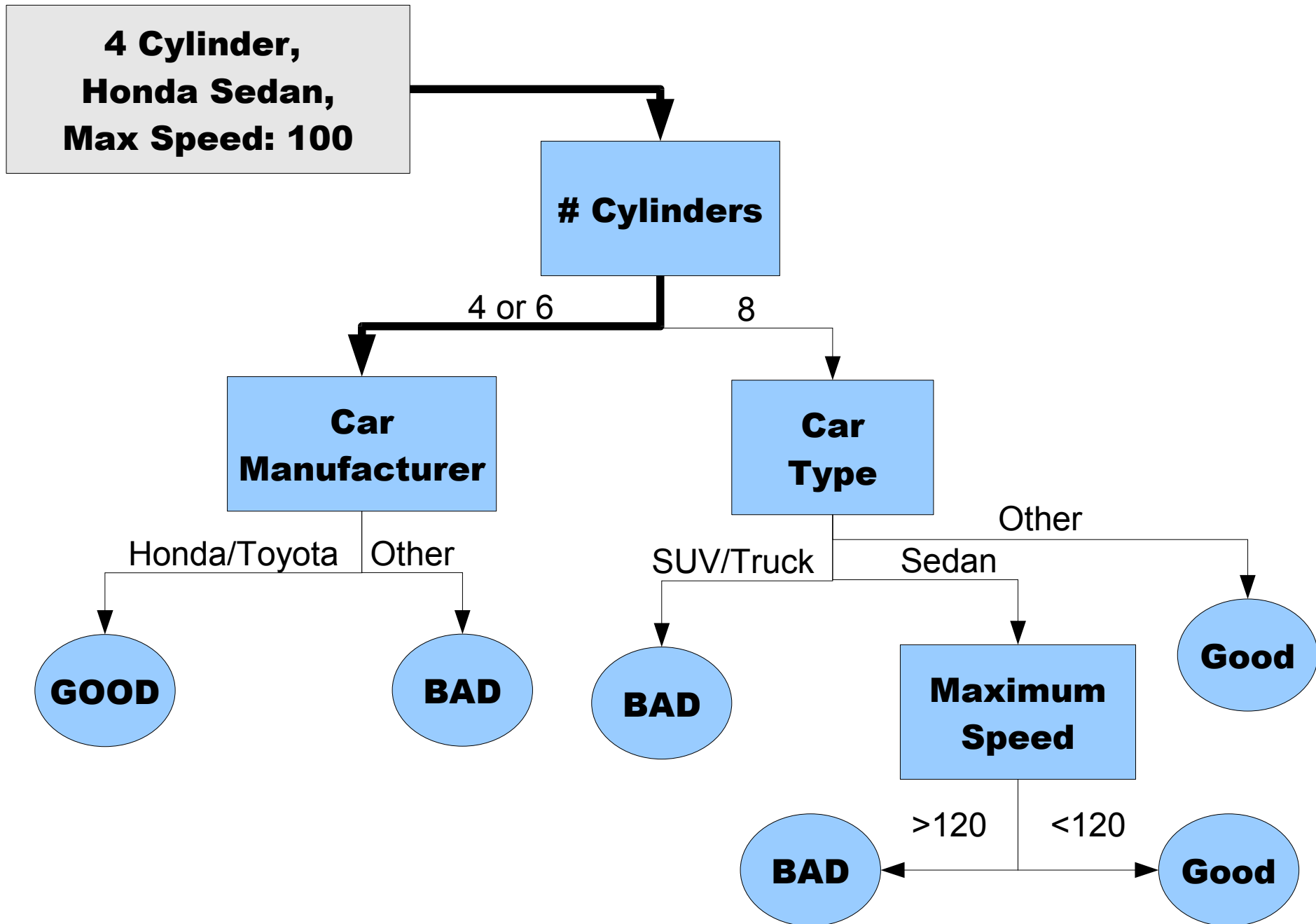
Decision Tree



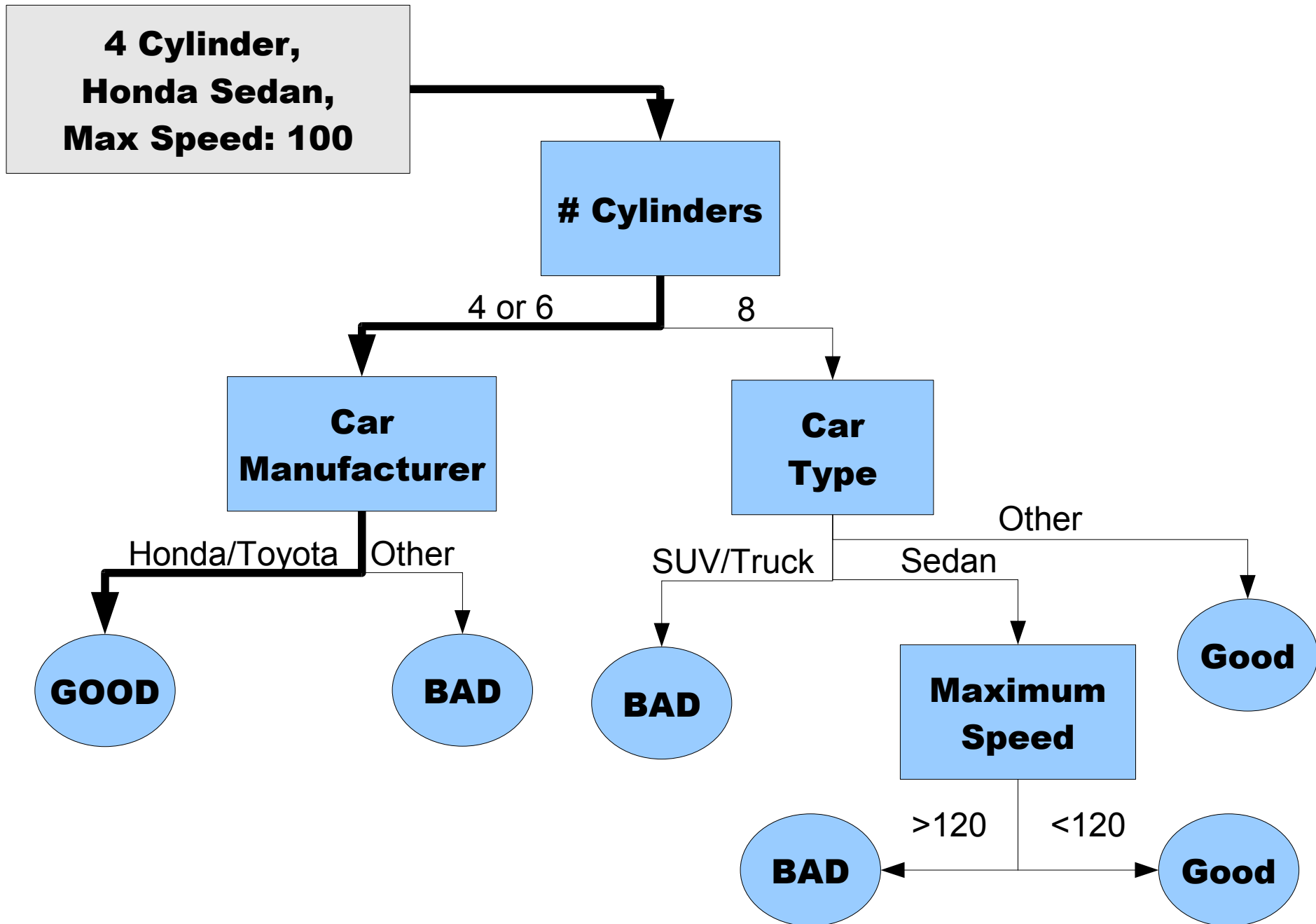
Decision Tree



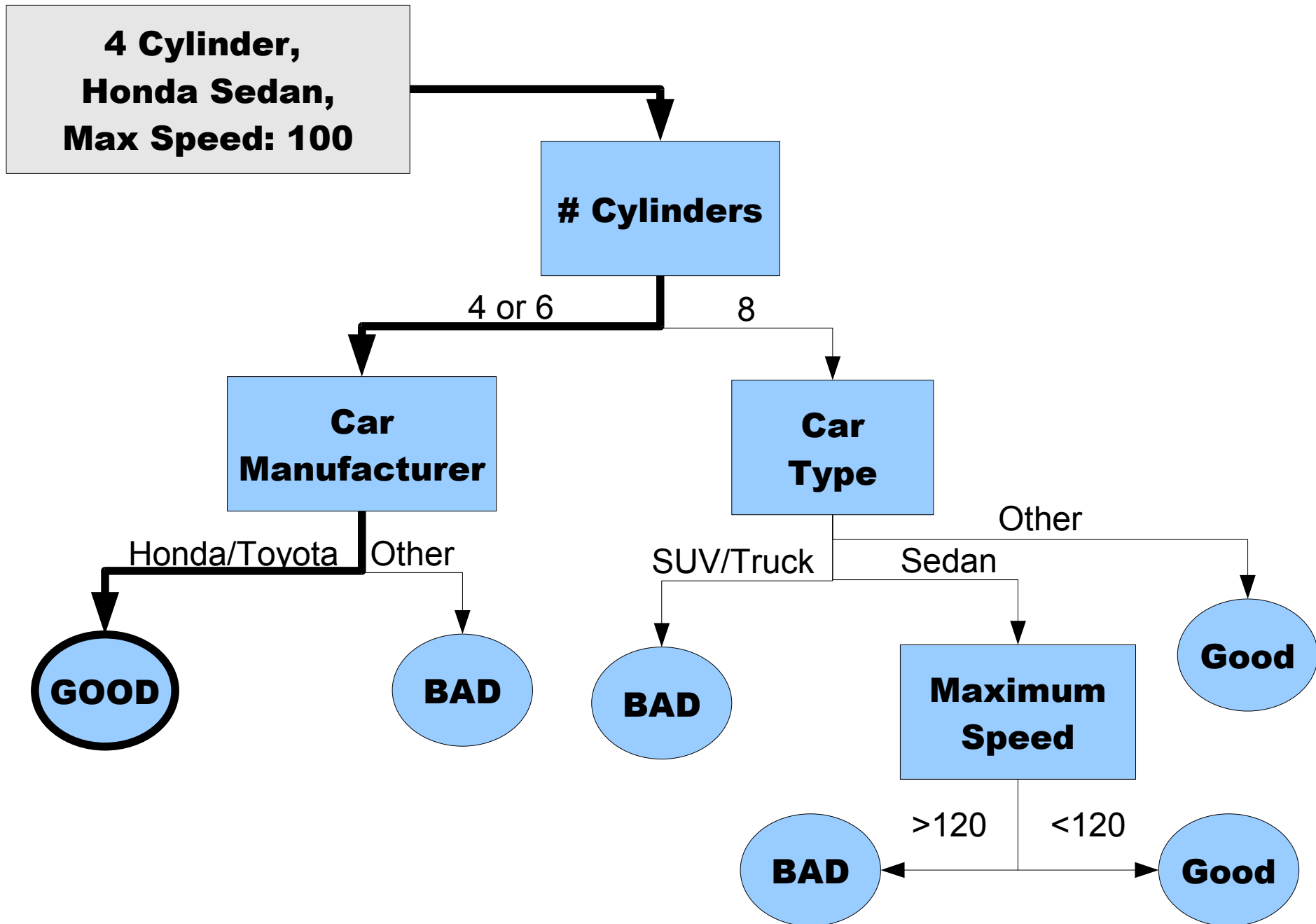
Decision Tree



Decision Tree

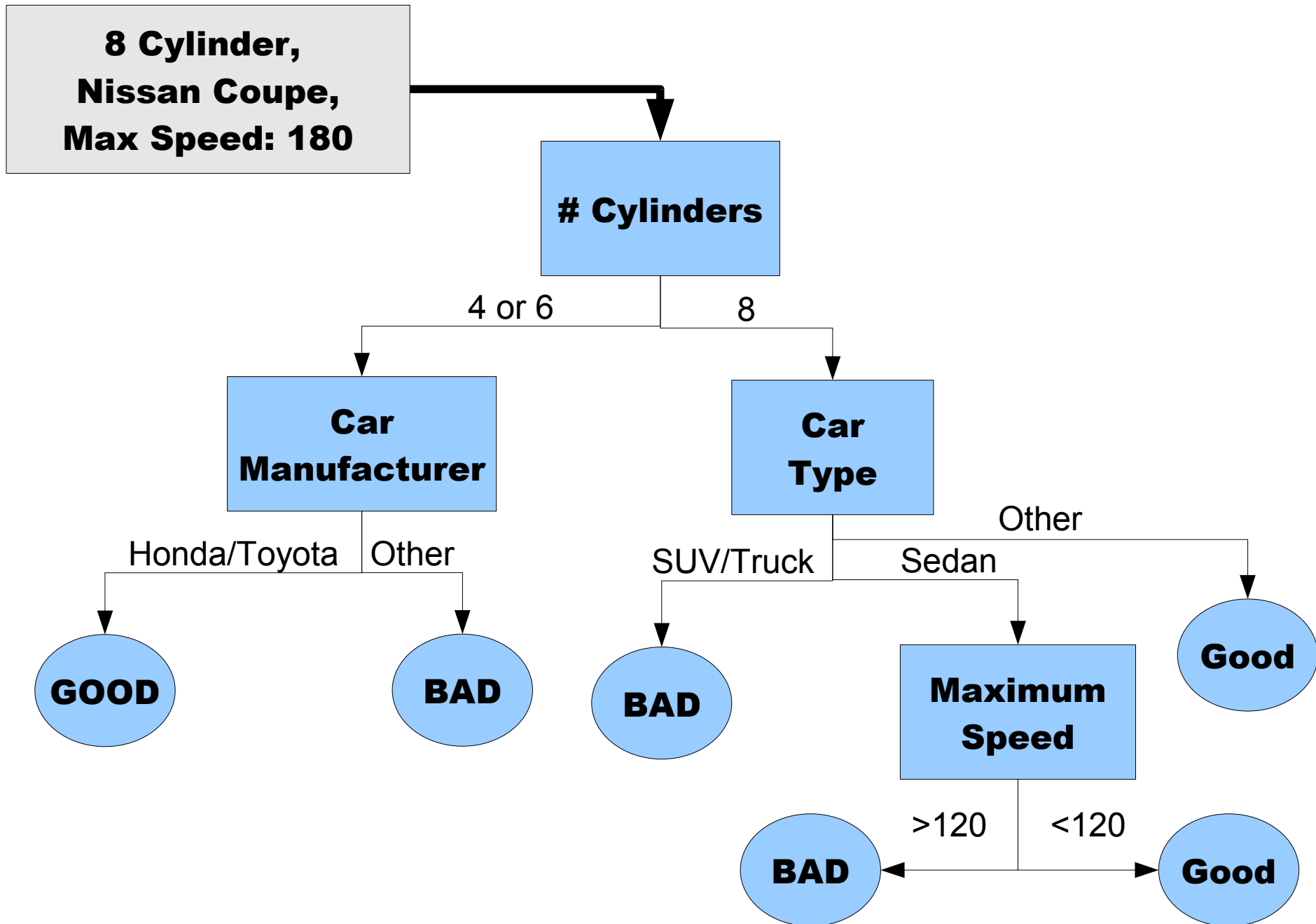


Decision Tree

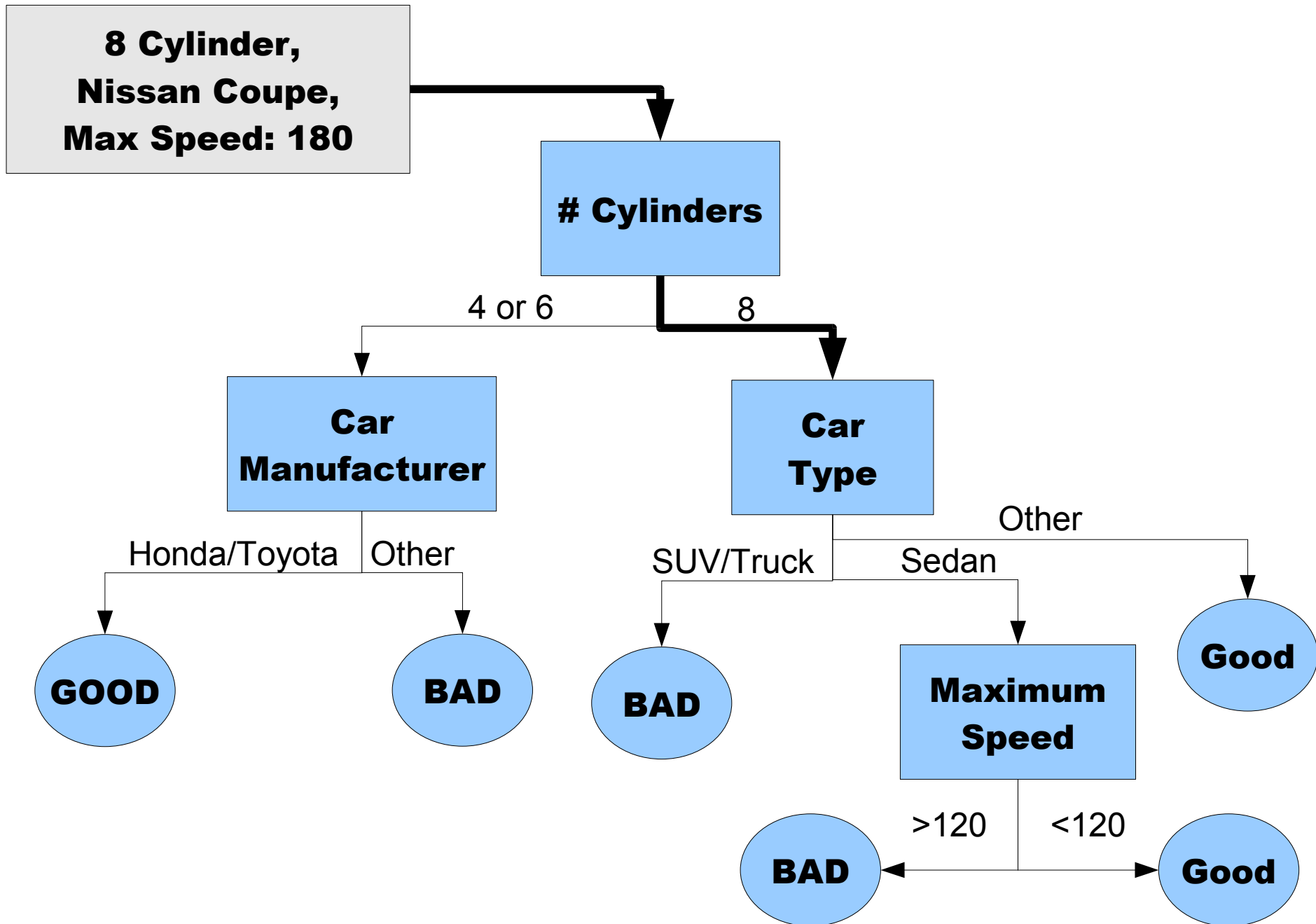


Another Example

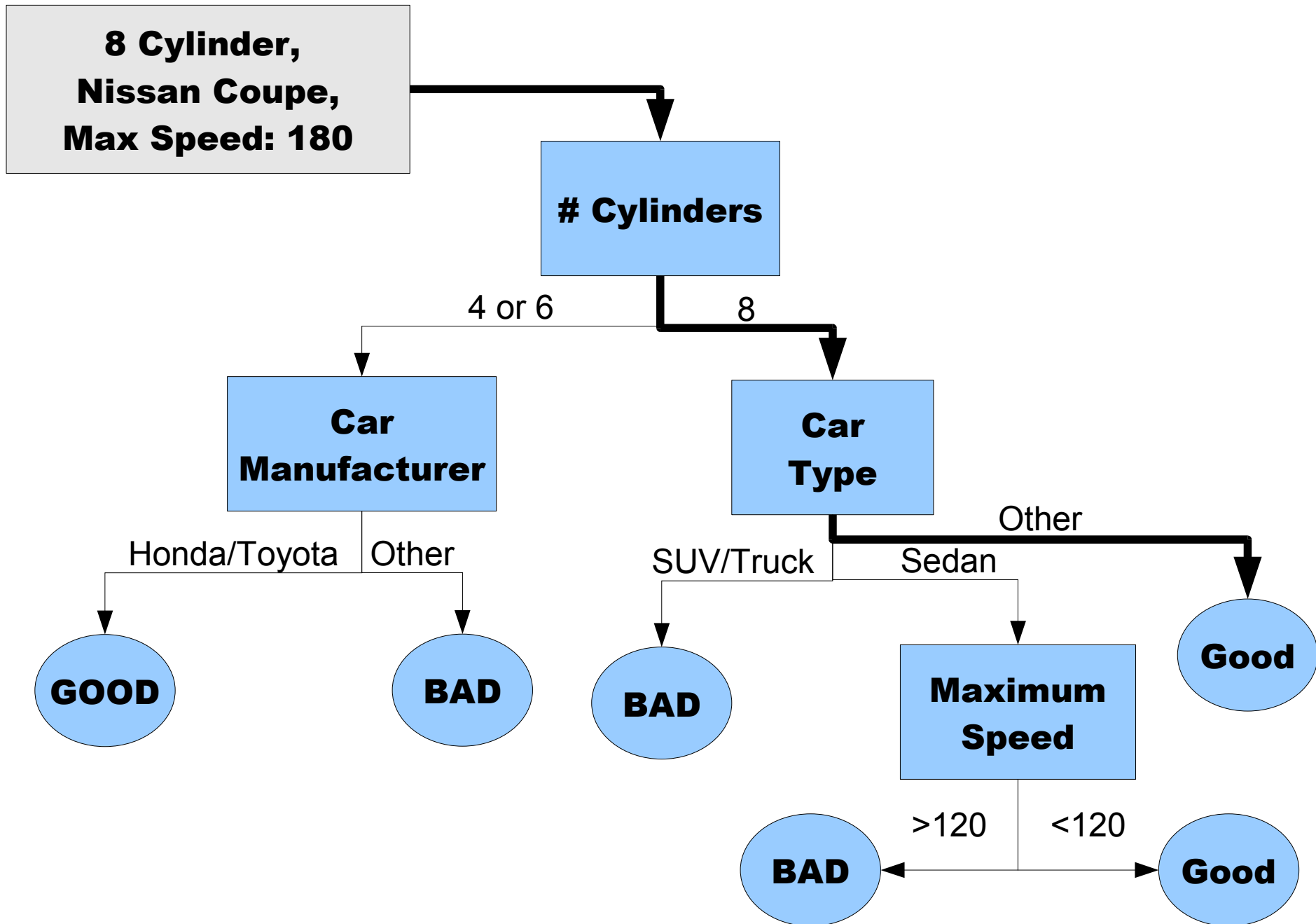
Decision Tree



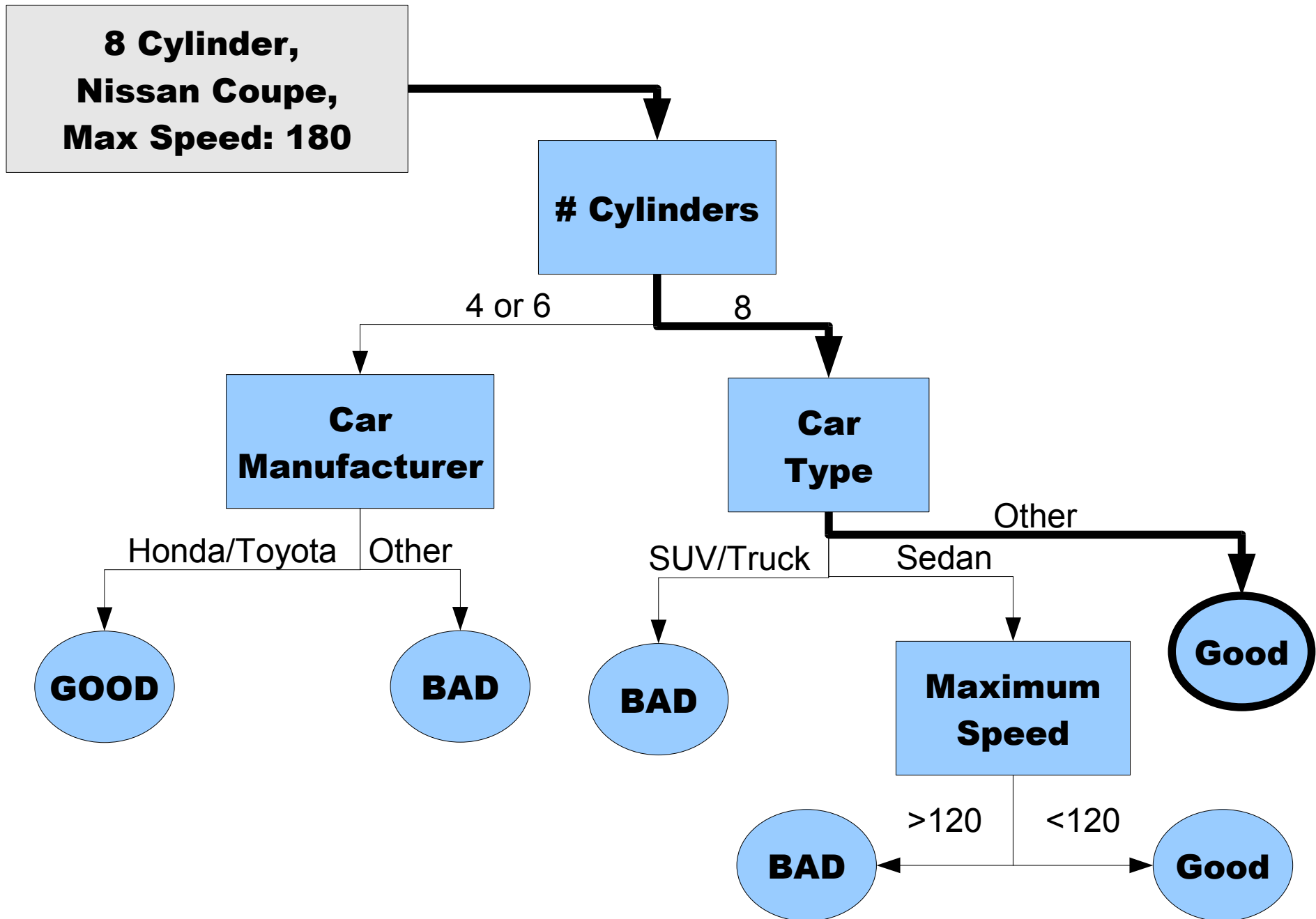
Decision Tree



Decision Tree



Decision Tree



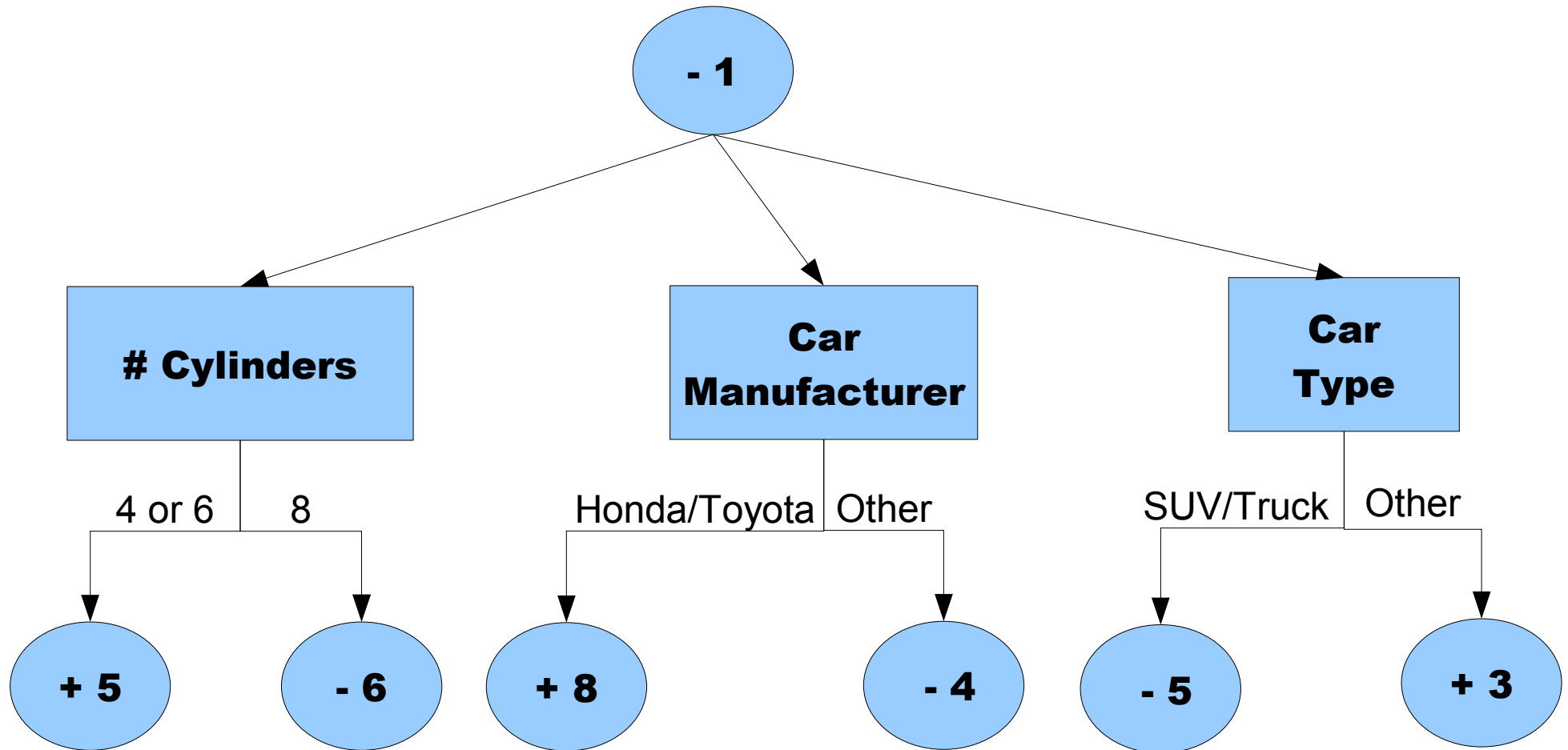
Decision Trees

- Follow a single path until reach decision
- No confidence levels
- Many criterion for growing decision trees

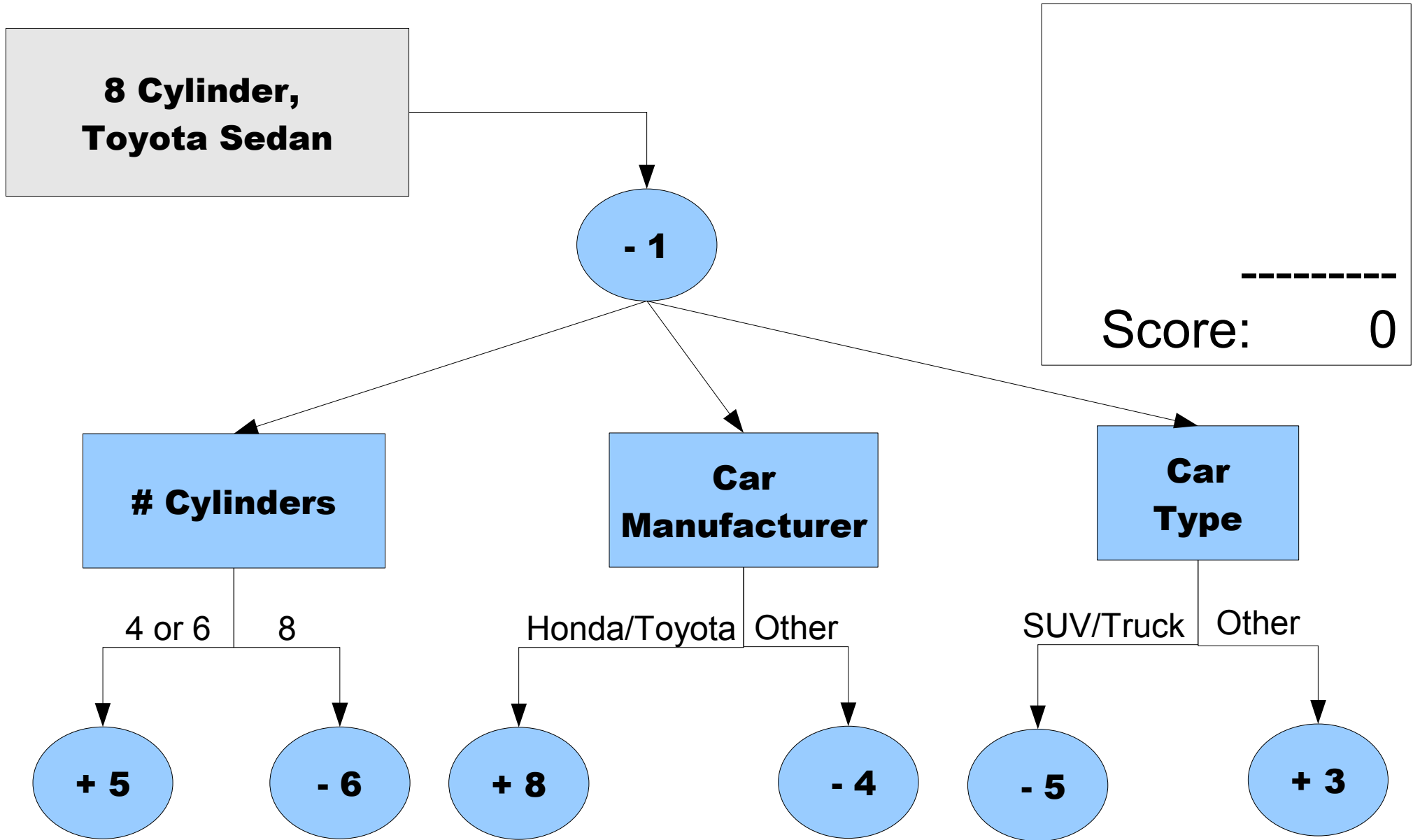
Final Classifier: Alternating Decision Tree

- Each path in tree is series of weak hypotheses
- Does the car have 4 or 6 cylinders?
 - Yes $\Rightarrow +5$, No $\Rightarrow -6$
- Is the car a Toyota or Honda?
 - Yes $\Rightarrow +8$, No $\Rightarrow -3$
- A Honda with 8 cylinders $\Rightarrow +2$

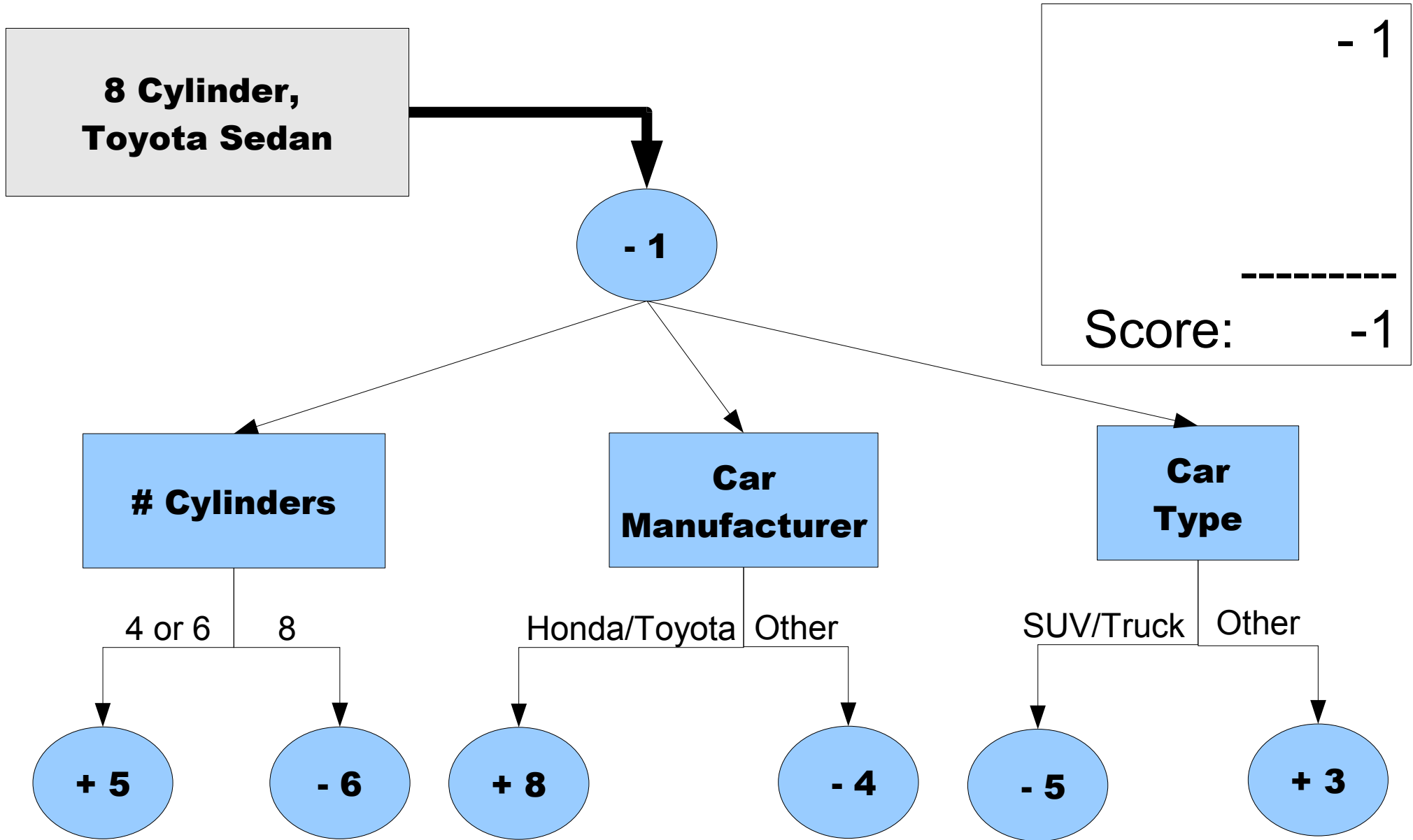
Alternating Decision Tree



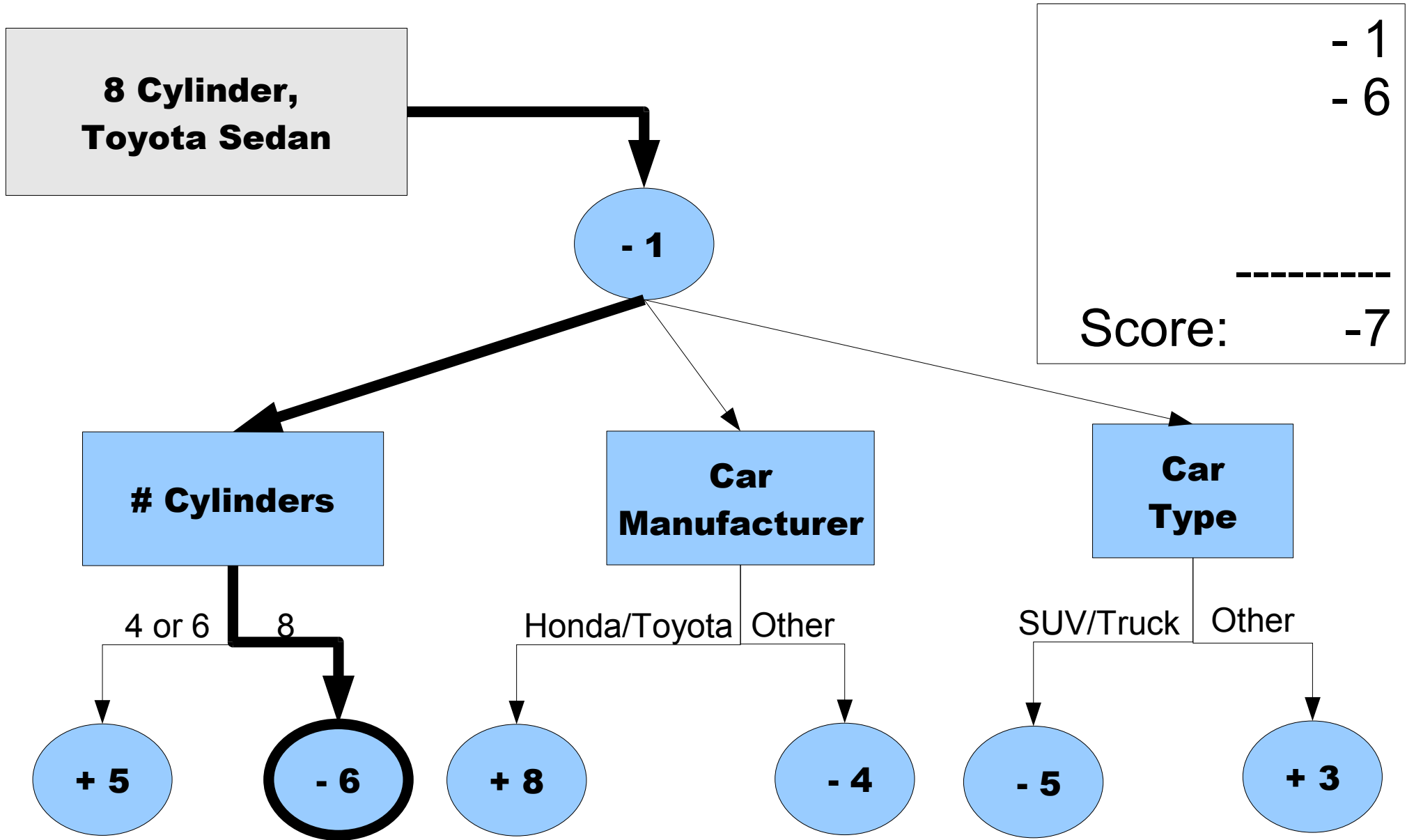
Alternating Decision Tree



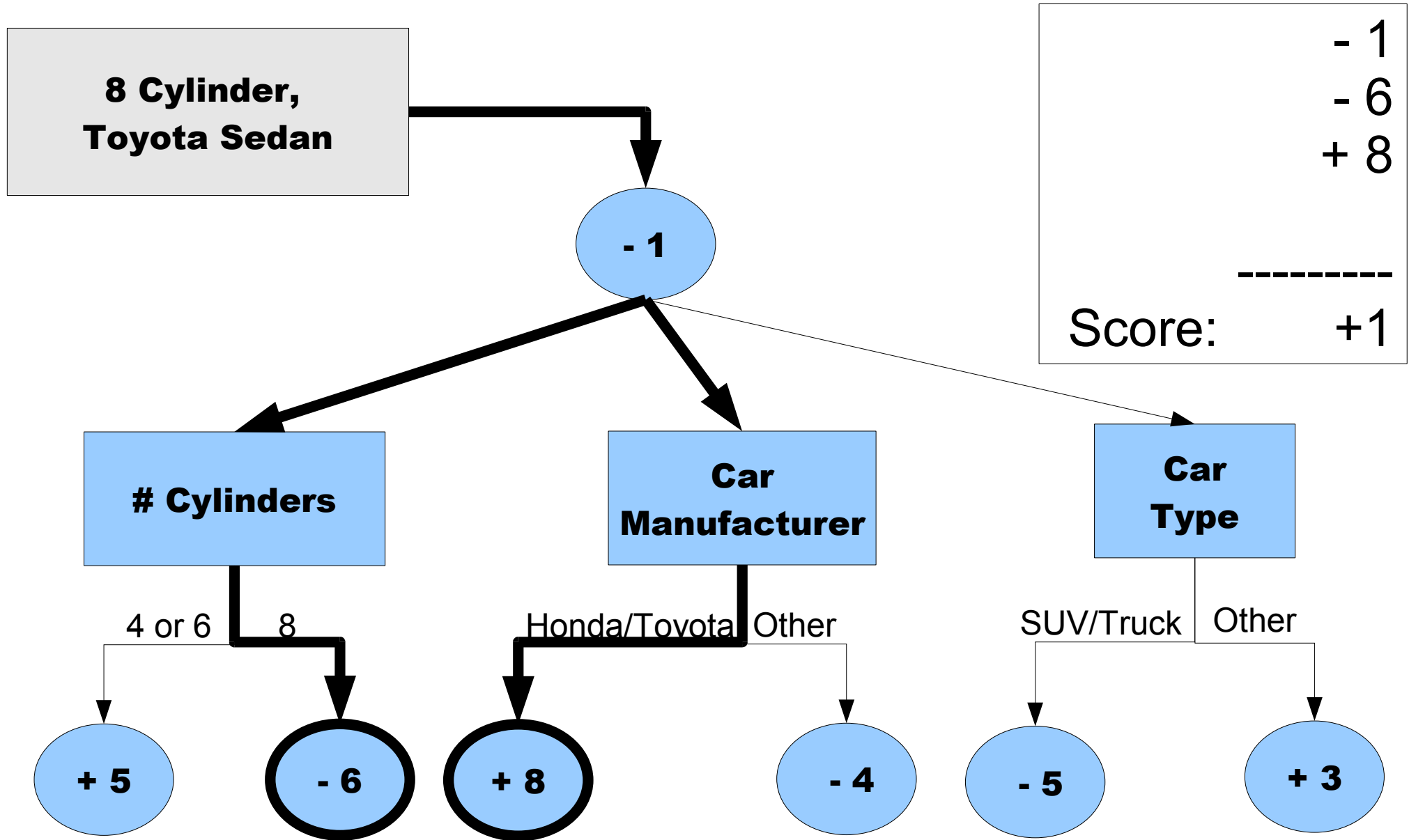
Alternating Decision Tree



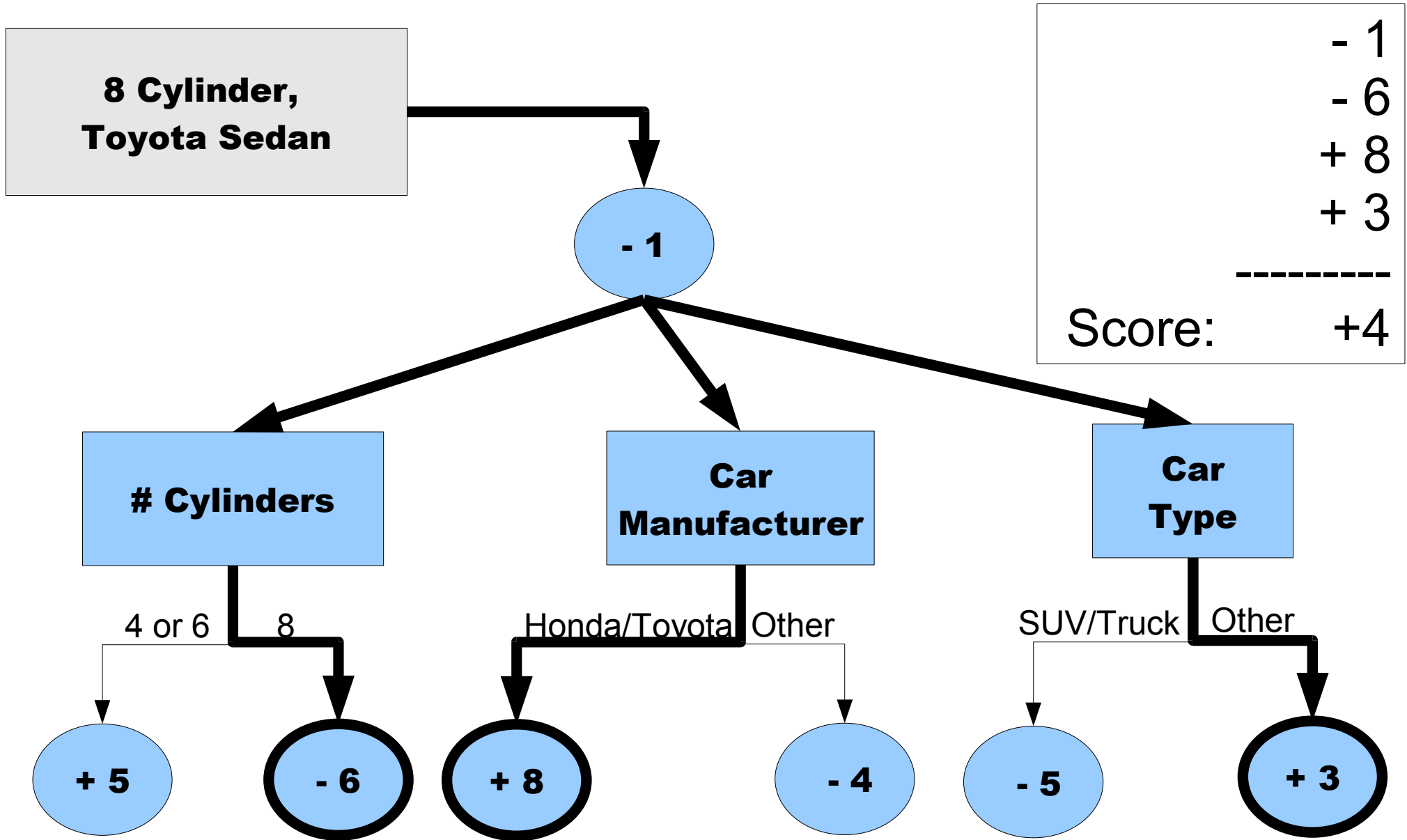
Alternating Decision Tree



Alternating Decision Tree



Alternating Decision Tree



Another Example

- Previous example was pretty simple
 - Just a series of decisions with weights
 - A basic additive linear model
- Next example shows a more interesting ATree
 - Has greater depth
 - Some weak hypotheses abstain
- Two inputs are shown

**8 Cylinder,
Nissan Sedan,
Max Speed: 180**

- 1

Cylinders

4 or 6

8

+ 5

- 6

Honda/Toyota

Other

+ 8

- 4

Max Speed

< 110

> 110

+ 2

- 3

Cylinders

4 or 6

8

- 1

+ 2

**Car
Manufacturer**

Car Type

SUV/Truck

Other

- 9

+ 7

Score: -----

- 1

- 1

**8 Cylinder,
Nissan Sedan,
Max Speed: 180**

- 1

Cylinders

**Car
Manufacturer**

4 or 6

8

Honda/Toyota

Other

+ 5

- 6

+ 8

- 4

Max Speed

Cylinders

Car Type

< 110

> 110

4 or 6

8

SUV/Truck

Other

+ 2

- 3

- 1

+ 2

- 9

+ 7

- 1
- 6

Score:

- 7

**8 Cylinder,
Nissan Sedan,
Max Speed: 180**

- 1

Cylinders

**Car
Manufacturer**

4 or 6

8

Honda/Toyota

Other

+ 5

- 6

+ 8

- 4

Max Speed

Cylinders

Car Type

< 110

> 110

4 or 6

8

SUV/Truck

Other

+ 2

- 3

- 1

+ 2

- 9

+ 7

- 1
- 6
- 4

Score: - 11

**8 Cylinder,
Nissan Sedan,
Max Speed: 180**

- 1

Cylinders

**Car
Manufacturer**

4 or 6

8

Honda/Toyota

Other

+ 5

- 6

+ 8

- 4

Max Speed

Cylinders

Car Type

< 110

> 110

4 or 6

8

SUV/Truck

Other

+ 2

- 3

- 1

+ 2

- 9

+ 7

- 1
- 6
- 4
+ 2

Score:

- 9

Another Example

**8 Cylinder,
Honda SUV,
Max Speed: 90**

- 1

Cylinders

**Car
Manufacturer**

Score: **- 1**

4 or 6

8

Honda/Toyota

Other

+ 5

- 6

+ 8

- 4

Max Speed

Cylinders

Car Type

< 110

> 110

4 or 6

8

SUV/Truck

Other

+ 2

- 3

- 1

+ 2

- 9

+ 7

**8 Cylinder,
Honda SUV,
Max Speed: 90**

- 1

Cylinders

**Car
Manufacturer**

Score: **- 7**

4 or 6

8

Honda/Toyota

Other

+ 5

- 6

+ 8

- 4

Max Speed

Cylinders

Car Type

< 110

> 110

4 or 6

8

SUV/Truck

Other

+ 2

- 3

- 1

+ 2

- 9

+ 7

**8 Cylinder,
Honda SUV,
Max Speed: 90**

- 1

Cylinders

**Car
Manufacturer**

- 1
- 6
+ 8

Score: + 1

4 or 6

8

Honda/Toyota

Other

+ 5

- 6

+ 8

- 4

Max Speed

Cylinders

Car Type

< 110

> 110

4 or 6

8

SUV/Truck

Other

+ 2

- 3

- 1

+ 2

- 9

+ 7

Another Example

**4 Cylinder,
Honda SUV,
Max Speed: 90**

- 1

Cylinders

**Car
Manufacturer**

Score: **- 1**

4 or 6

8

Honda/Toyota

Other

+ 5

- 6

+ 8

- 4

Max Speed

Cylinders

Car Type

< 110

> 110

4 or 6

8

SUV/Truck

Other

+ 2

- 3

- 1

+ 2

- 9

+ 7

**4 Cylinder,
Honda SUV,
Max Speed: 90**

- 1

Cylinders

**Car
Manufacturer**

**- 1
+ 5**

**Score: -----
 + 4**

4 or 6 8

Honda/Toyota Other

+ 5

- 6

+ 8

- 4

Max Speed

Cylinders

Car Type

< 110 > 110

4 or 6 8

SUV/Truck Other

+ 2

- 3

- 1

+ 2

- 9

+ 7

**4 Cylinder,
Nissan SUV,
Max Speed: 90**

- 1

Cylinders

**Car
Manufacturer**

- 1
+ 5
+ 2

Score: + 6

4 or 6 8

+ 5

- 6

+ 8

- 4

Honda/Toyota Other

Max Speed

Cylinders

Car Type

< 110 > 110

4 or 6 8

SUV/Truck Other

+ 2

- 3

- 1

+ 2

- 9

+ 7

**4 Cylinder,
Nissan SUV,
Max Speed: 90**

- 1

Cylinders

**Car
Manufacturer**

4 or 6 8

Honda/Toyota Other

+ 5

- 6

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- 4

Max Speed

Cylinders

Car Type

< 110 > 110

4 or 6 8

SUV/Truck Other

+ 2

- 3

- 1

+ 2

- 9

+ 7

- 1
+ 5
+ 2
- 4

Score: -----
+ 2

**4 Cylinder,
Nissan SUV,
Max Speed: 90**

- 1

Cylinders

**Car
Manufacturer**

4 or 6

8

Honda/Toyota

Other

+ 5

- 6

+ 8

- 4

Max Speed

Cylinders

Car Type

< 110

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SUV/Truck

Other

+ 2

- 3

- 1

+ 2

- 9

+ 7

- 1
+ 5
+ 2
- 4
- 1
+ 7

Score:

+ 8

ATree Pros and Cons

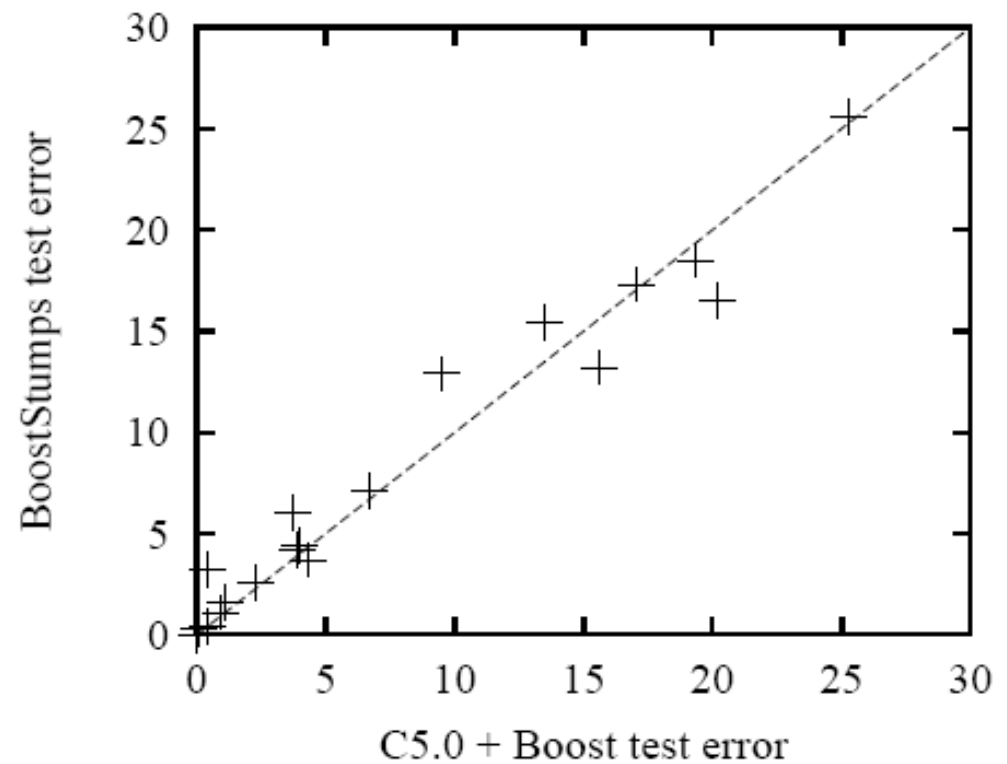
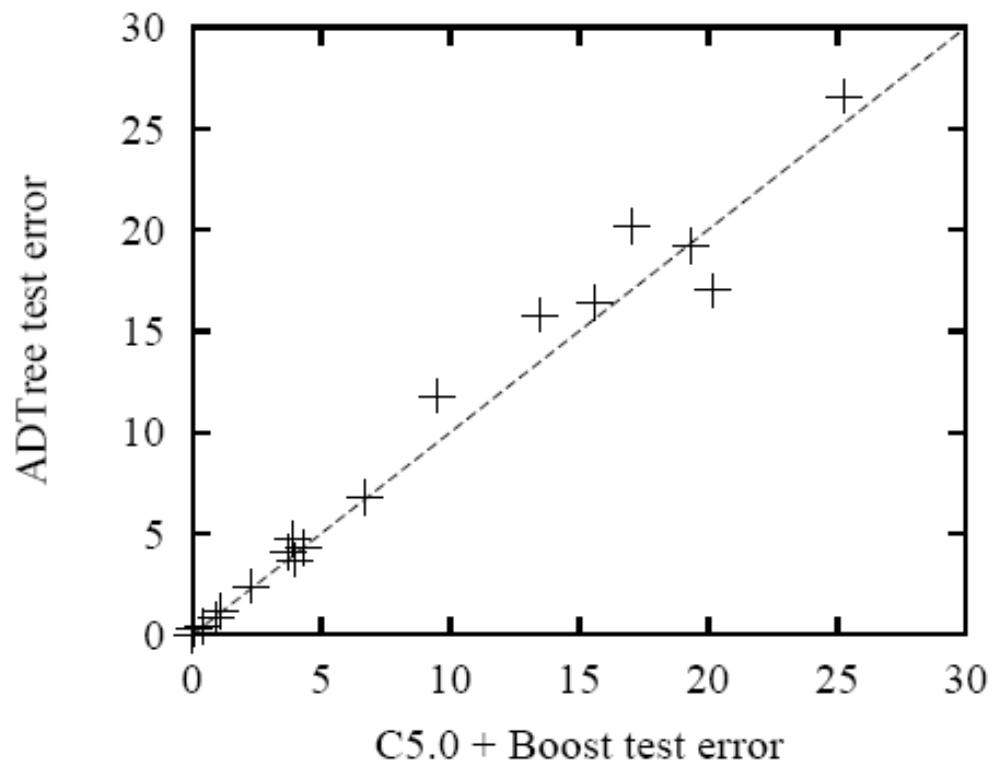
Pros

- Can focus on specific regions
- Similar test error to other boosting methods
- Requires far fewer iterations
- Easily visualizable

Cons

- Larger VC-dimension
 - Increased proclivity for overfitting

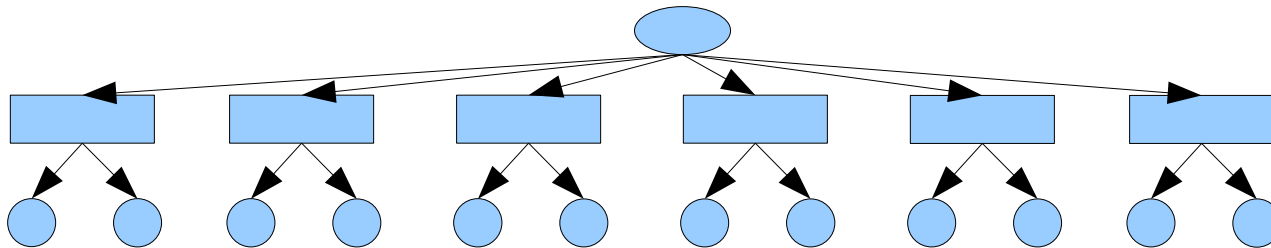
Error Rates



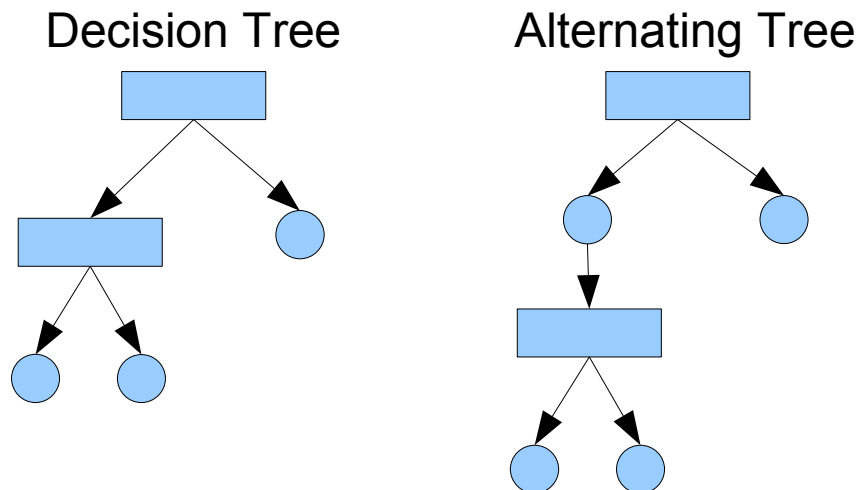
Taken from Freund & Mason 1997

Some Basic Properties

- ATrees can represent decision trees, boosted decision-stumps, and boosted decision trees
- ATrees for boosted decision stumps:



- ATrees for decision trees:



Resources

- Boosting.org
- JBoost software available at <http://www.cs.ucsd.edu/users/aarvey/jboost/>
 - Implementation of several boosting algorithms
 - Uses ATrees as final classifier
- Rob Schapire keeps a fairly complete list <http://www.cs.princeton.edu/~schapire/boost.html>
- Wikipedia